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An input variable partitioning algorithm for functional decomposition of a system of Boolean functions based on the tabular method

Saeid Taghavi Afshord [a,*], Yuri Pottosin [b], Bahman Arasteh [c]

[a] Department of Computer Engineering, Shabestar Branch, Islamic Azad University, Shabestar, Iran
[b] United Institute of Informatics Problems of the NAS of Belarus, Minsk, Belarus
[c] Department of Computer Engineering, College of Engineering, Tabriz Branch, Islamic Azad University, Tabriz, Iran

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ABSTRACT

Functional decomposition is a fundamental method for the optimization of multi-level logic circuits by breaking down a complex circuit into smaller and hopefully simpler sub-circuits. The problem of searching for an appropriate partition of input variables is the first step in the decomposition process, and it is a challenging task in the logic synthesis. The serial, two-block disjoint decomposition of a system of completely specified Boolean functions is investigated. An input variable partitioning algorithm is proposed, which is used for functional decomposition along with a tabular method. It is based on using the ternary matrix cover approach. A method for constructing the cover map that has the structure of the Karnaugh map is also presented. It is used in a step of the tabular method during the decomposition. There is a chance in a decomposable system, to choose such a solution of the task to reduce the circuit size exponentially. The paper emphasizes on obtaining the best possible partition in an efficient manner. A set of experiments has been carried out on the generated systems of Boolean functions and standard benchmarks. The results confirm the efficiency and effectiveness of the suggested algorithm along with the ternary matrix cover approach. The obtained solutions are optimal in the most cases, according to a certain criterion.

1. Introduction

The problem of decomposition of Boolean functions is one of the most important problems of logical design. Decomposition breaks a function block into smaller function blocks. It is performed in such a way that the original system behavior is perceived, and each of the decomposed subsystems is simpler than the original system to analyze, realize and synthesize [14]. It is important to find a successful solution for this problem, because it has a direct influence on the quality and cost of digital devices at the design process. It has been shown in [8,18] that a considerable number of papers have been already published on this topic and it is still interesting for the research [21,16,28,6,14,23,19]. The motivation is reducing the complexity of the problem by divide and conquer approach to find an appropriate set of sub-circuits. Efficiency and

* Corresponding author.
E-mail addresses: taghavi@iaushab.ac.ir (S. Taghavi Afshord), pott@newman.bas-net.by (Y. Pottosin), b_arasteh@iaut.ac.ir (B. Arasteh).

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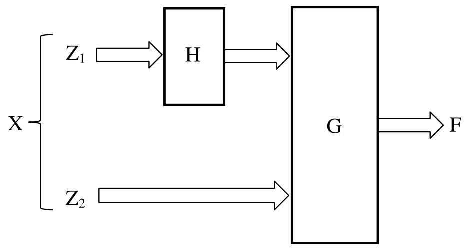

Fig. 1. Schematic representation of functional decomposition.

usefulness of functional decomposition have been demonstrated in theoretical papers [2,9,16,23]. It can be applied in many fields of modern engineering and science, such as combinational and sequential logic synthesis for VLSI systems, machine learning, decision systems, databases, data mining, etc. [22,15,23].

Searching for a solution of the input variable partitioning task is regarded as an NP-hard problem, because it is equivalent to the well-known set-covering problem [18,22]. Therefore, several heuristic methods have been proposed in the literature to avoid an exhaustive search [21,16,22]. Although, heuristic approaches cannot guarantee the quality of the obtained results, sometimes they are the only way to solve the problem.

An appropriate partition can be considered from different point of views. When the number of the input variables of a given system of Boolean functions is low and the system is decomposable, the best partition is found from the circuit size perspective. An algorithm for the bound set selection close to the half of the arguments of the given system is presented. It can decrease the circuit size that induced from decomposition. The number of disjunctive normal forms (DNF terms) is decreased in the obtained superposition, and consequently the area of the programmable logic array (PLA) is reduced. The main advantage of this algorithm is reducing the search space of the exponentially growing partitions, in the most cases. The ternary matrix cover approach [20] is used to determine the decomposability of a given system of Boolean functions and to acquire the best solution, simultaneously. By using a compact table, the existence of a solution of the problem can be proved rather easily, and if it does exist, the corresponding superposition is simply found.

The paper is organized as follows: Section 2 describes the definition of the problem, and basic concepts in details that are used in the sequel. It introduces the ternary matrix cover approach for decomposition of Boolean functions, as well as a new method to obtain the cover of the ternary matrix is described. Section 3 deals with the partitioning problem according to a certain criterion. The proposed algorithm along with the ternary matrix cover approach is given. In Section 4, the results of the implemented method in finding the best solution are demonstrated, and they are compared with existing methods. Section 5 concludes the paper.

2. Definitions and preliminary stages

A function $F(X)$ may be expressed using other functions, G and H, as follows: $F(X) = G(H(Z_1), Z_2)$ where $Z_1 \cup Z_2 = X$. If such a representation exists, it is called a *functional decomposition* of F. Generally, G and H are less complex than F. The subsets Z_1 and Z_2 are called bound and free sets, respectively. A few papers deal with the search for the partition $\{Z_1, Z_2\}$ at which this problem has a solution [2,9,18,29,21,16,7,19]. In this paper, the main attention is paid to the search for subsets Z_1 and Z_2 in a particular way. The general scheme of the functional decomposition is illustrated in Fig. 1. Furthermore, if the function H has only one output, it is called a simple decomposition.

We now formally define the problem in the matrix representation used in the sequel. Let a system of completely specified Boolean functions $\boldsymbol{y} = \boldsymbol{f}(\boldsymbol{x})$, where $\boldsymbol{y} = (y_1, y_2, \ldots, y_m)$, $\boldsymbol{x} = (x_1, x_2, \ldots, x_n)$ and $\boldsymbol{f}(\boldsymbol{x}) = (f_1(\boldsymbol{x}), f_2(\boldsymbol{x}), \ldots, f_m(\boldsymbol{x}))$, be given by matrices $\boldsymbol{U}$ and $\boldsymbol{V}$ that are the matrix representation of the system of DNFs of the given functions [32]. Matrix $\boldsymbol{U}$ is a ternary matrix of dimension $l \times n$, where l is the number of terms in the given DNFs. The columns of $\boldsymbol{U}$ are marked with the variables $x_1, x_2, \ldots, x_n$, and the rows represent the terms of the DNFs (the intervals of the space of the variables $x_1, x_2, \ldots, x_n$). The matrix $\boldsymbol{V}$ is a Boolean matrix of dimension $l \times m$, and its columns are marked with the variables $y_1, y_2, \ldots, y_m$. The ones in these columns point out the terms in the given DNFs. A row $\boldsymbol{u}$ in $\boldsymbol{U}$ *absorbs* a Boolean vector $\boldsymbol{a}$ if $\boldsymbol{a}$ belongs to the interval represented by $\boldsymbol{u}$.

2.1. Setting the problem

The task is considered as follows: In a given system of completely specified Boolean functions $\boldsymbol{y} = \boldsymbol{f}(\boldsymbol{x})$, the superposition $\boldsymbol{y} = \boldsymbol{\varphi}(\boldsymbol{w}, \boldsymbol{z}_2)$, $\boldsymbol{w} = g(\boldsymbol{z}_1)$ should be found, where $\boldsymbol{z}_1$ and $\boldsymbol{z}_2$ are vector variables whose components are Boolean variables in the subsets Z_1 and Z_2 of the set $X = \{x_1, x_2, \ldots, x_n\}$, respectively such that $X = Z_1 \cup Z_2$ and $Z_1 \cap Z_2 = \varnothing$. The number of components of the vector variable $\boldsymbol{w}$ must be less than that of $\boldsymbol{z}_1$. Such a kind of decomposition is called two-block disjoint decomposition [8,32].