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The deterministic product location problem under a pick-by-order policy

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ABSTRACT

This paper treats the product location problem in warehouses, i.e., stock keeping units (SKUs) are to be assigned to storage positions in order to minimize the resulting picking effort when retrieving SKUs in a pick-by-order environment. We restrict our view on warehouses having a single cross aisle and show that already very simple layouts consisting of only a single rack lead to NP-hard optimization problems. In addition to a complexity analysis for different layouts, elementary solution procedures are introduced and tested. Finally, we investigate the robustness of our deterministic problem when facing erroneous input data.

1. Introduction

Order picking is an elementary warehousing function dealing with the retrieval of stock keeping units (SKUs) from their storage locations in order to satisfy a given demand defined by picking orders. Typically, order picking is a labor-intensive task, which is both time-critical and cost-intensive. Therefore, it is not astounding that optimizing picking operations by computerized solution procedures has received (and is still receiving) plenty of attention among practitioners and academic researchers alike (e.g., see the latest survey papers on warehouse operations by de Koster et al. [5]; Gu et al. [10]). In this context, we consider the deterministic product location problem (also known as the slotting problem), which is defined as follows:

Consider a given set $P = \{1, \dots, n\}$ of SKUs to be assigned to a given set of storage locations $S = \{s_1, \dots, s_n\}$ arranged in racks of a given layout, so that each product receives exactly one storage location. Once SKUs are assigned they need to be picked according to a given set O containing m picking orders, where each order $o \in O$ consists of a subset of SKUs to be retrieved (e.g., all items ordered by a specific customer). Order picking is organized according to the pick-by-order policy (also denoted as discrete order picking), so that all SKUs of an order are jointly picked during a single tour of a picker. Capacity constraints are considered to be a non-issue, so that exactly m tours occur. A tour starts and ends in the depot (or input–output point) and consists of an ordered set of locations where the products of the respective order are stored. We restrict our view on warehouse layouts having only a single front-end cross-aisle, i.e., for reaching another storage aisle, so that pickers enter and leave storage aisles over the same entrance point at the front (also denoted as the "out and back" approach or the "return policy"). Note that our (front-end) cross-aisle should not be confound with a middle aisle crossing the storage area in order to introduce additional flexibility for the pickers changing storage aisles. We abstain from the latter type of (middle) cross-aisle, so that we restrict ourselves to what is also denoted as the one-block layout. Both assumptions, the one-block layout plus the "out and back" policy, make the integrated picker routing problem trivial to solve, i.e., only the

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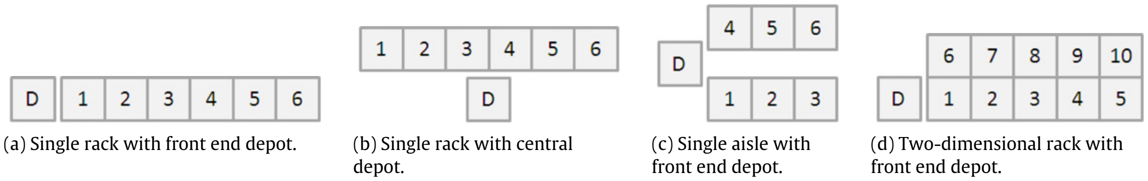

Fig. 1. Investigated layouts for the storage location problem (PLP) ((a)–(c) top view and (d) side view).

farthest picking location per order and aisle has to be determined. Within this setting, our product location problem (PLP) aims to assign products to storage locations, so that the total walking distance over all given picking orders is minimized.

This paper investigates the computational complexity of the PLP in different warehouse layouts and presents suited solution procedures. Specifically, we investigate the most basic PLP-layout (depicted in Fig. 1(a)) consisting of a single straight rack and a front end depot. For this building block we prove NP-hardness in the strong sense and present exact and heuristic optimization procedures. Then, we show how these basic results can be applied for proving computational complexity for other layouts, i.e., layouts (b)–(d) in Fig. 1, and how the solution procedures can successfully be applied for repeatedly solving subproblems in real-world order picking settings.

The remainder of the paper is structured as follows. Section 2 provides a literature review on related research. Then, Section 3 investigates layout (a) where a single rack faces a front end depot. For this basic PLP, a complexity analysis is provided and exact and heuristic solution procedures are introduced, which are tested in a comprehensive computational study. The complexity results are then applied for proving NP-hardness of more complex layouts, i.e., layouts (b)–(d), within Section 4. Furthermore, we investigate a typical real-world setting for order picking and show our basic PLP being an important subproblem (Section 5). Section 6 investigates the robustness of PLP solutions if incomplete or erroneous order information is available and, finally, Section 7 concludes the paper.

2. Literature review

Numerous studies on optimizing warehouse operations in general and on product location and picker routing in particular exist. Instead of trying to summarize this vast body of literature we refer to the excellent survey papers published on these topics over the past years, e.g., [5,9,19,10]. To emphasize the importance of simultaneously planning product locations and picker routes, we only make one exception. In a recent case study at a distribution center for alcoholic beverages in Québec (Canada), Renaud and Ruiz [18] calculate an impressive reduction rate of 27% on the total picking effort when unifying both problems. With regard to the information availability, i.e., the question whether reliable information on (short term) picking orders exist when solving the (long term) product location problem, three cases can be distinguished:

- Whenever the composition of orders considerably varies over time, e.g., induced by highly volatile customer demands, orders are hardly predictable and *no reliable order information* is available. In this case, only product specific information is on hand, so that, for instance, the famous cube-per-order index (see [12]) can be applied for assigning storage positions to SKUs.
- In a more stable environment, reliable *product correlations* defining the probability of a product pair being jointly ordered may be obtained from historical data (see [7,3]). These correlations allow for a grouping of product families, so that similar SKUs are located in the same region of the storage area and travel distance during operational order picking can be reduced.
- This paper assumes a given *deterministic order set*. Clearly, this assumption seems unrealistic for volatile environments faced by most distribution centers serving demands of final customers. However, a given order set may be a valid approximation of reality in intermediate warehouses, where, for instance, recurrent orders for parts consumed by cyclically produced production lots in a low-variety make-to-stock supply chain are picked. Even in a truly stochastic environment the deterministic problem can contribute to the overall solution, e.g., in a scenario-based approach. Furthermore, solving the PLP with a deterministic order set can deliver benchmark solutions for the former two cases, in order to quantify the value of information from a theoretical point of view.

To the best of the authors' knowledge there exists only the study of Van Oudheusden and Zhu [20], who also treat a product location problem for a given set of recurrent orders. They briefly mention the basic PLP resulting from layout (a) in Fig. 1 as an interesting subproblem and present a very basic myopic procedure. A complexity analysis and exact solution procedures for our PLP are not provided. Instead, they focus a more complex layout with a two-dimensional rack and a front end depot (layout (d) in Fig. 1) applied in a man-aboard automated storage and retrieval system. They presuppose given routes for each order predetermined by some traveling salesman procedure, so that the routing problem is excluded, and provide a heuristic solution procedure for the product location problem. Up to now, the complexity status of this PLP has been unknown. In Section 4.3 we finally prove the PLP for layout (d) being strongly NP-hard.

With regard to its mathematical structure our basic PLP bears some similarities to the single-row equidistant facility layout problem (e.g., [14,15]), where facilities (e.g., are to be assigned to locations (equally spaced along a straight line)), so that the sum of product flows between facilities weighted with the distances of their assigned locations is minimized.