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Restricted power domination and fault-tolerant power domination on grids[☆]

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ABSTRACT

The power domination problem is to find a minimum placement of phase measurement units (PMUs) for observing the whole electric power system, which is closely related to the classical domination problem in graphs. For a graph $G = (V, E)$, the power domination number of G is the minimum cardinality of a set $S \subseteq V$ such that PMUs placed on every vertex of S results in all of V being observed. A vertex with a PMU observes itself and all its neighbors, and if an observed vertex with degree $d > 1$ has only one unobserved neighbor, then the unobserved neighbor becomes observed. Although the power domination problem has been proved to be NP-complete even when restricted to some special classes of graphs, Dorfling and Henning in [M. Dorfling, M.A. Henning, A note on power domination in grid graphs, Discrete Applied Mathematics 154 (2006) 1023–1027] showed that it is easy to determine the power domination number of an $n \times m$ grid. Their proof provides an algorithm for giving a minimum placement of PMUs. In this paper, we consider the situation in which PMUs may only be placed within a restricted subset of V. Then, we present algorithms to solve this restricted type of power domination on grids under the conditions that consecutive rows or columns form a forbidden zone. Moreover, we also deal with the fault-tolerant measurement placement in the designed scheme and provide approximation algorithms when the number of faulty PMUs does not exceed 3.

1. Introduction

As an important application in energy management, electric power companies use available measurements to continually monitor their system's state defined by a set of state variables (such as bus voltage magnitudes at loads and machine phase angles at generators [13]). The measured data are gathered by devices called phase measurement units (abbreviated as PMUs) and made available to estimate the system state. To achieve high accuracy in this estimation, a solution requires the system network to be observable. A system is said to be *observed* if all of the state variables of the system can be inspected by a set of PMUs. Thus, the system observability is determined by the number of PMUs and their deployment in the network. Although usually increasing the number of PMUs contributes to the system observability, the locations where PMUs are placed are also essential. It is often desirable to build a fully observable system with the minimum possible cost. A well-designed measurement placement can possibly make the whole system observable using fewer PMUs and thus reduce the

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overall cost. Therefore, designing a satisfactory measurement placement scheme has become an important issue and is widely studied in [3,5,13–16].

As usual, an electric power system is represented by an undirected graph $G = (V, E)$, where V is a set of vertices containing all electric nodes of the system, and E is a set of edges containing all transmission lines joining electric nodes. A PMU measures the state variable of the vertex at which it is placed and observes its incident edges and their endvertices. More precisely, we have the following rules defined in [8]:

1. Any vertex that is incident to an observed edge is observed.
2. Any edge joining two observed vertices is observed.
3. If a vertex is incident to a total of $k > 1$ edges and if $k - 1$ of these edges are observed, then all k of these edges are observed.

The problem considered here is the placement of a minimum set of PMUs, so that the system is topologically observable.

Haynes et al. [8] first considered the graph theoretical representation of the power system monitoring problem as a variation of the well-known graph domination problem (see [9,10]). For a graph $G = (V, E)$, a set $S \subseteq V$ is said to be a *power dominating set* (abbreviated as PDS) of G if every vertex and every edge in G is observed by S. The *power domination number* of G, denoted by $\gamma_P(G)$, is the minimum cardinality of a PDS of G. A PDS of G with the minimum cardinality is called a $\gamma_P(G)$-set, and the *power domination problem* is the problem of finding such a $\gamma_P(G)$-set. Haynes et al. [8] showed that the power domination problem is NP-complete even when restricted to some special classes of graphs such as bipartite graphs or chordal graphs. For recent results related to power domination on graphs, the reader can also refer to [1,2,5–8,11,12,17,18].

In this paper, we consider the situation in which PMUs may only be placed within a restricted subset of V and the case that the fault-tolerant measurement placement is involved in the designed scheme. The former situation is due to considerations of cost saving, security policy, convenience of installation, and other factors, while the latter case is to maintain the ability of measurement when an emergency is caused by faulty PMUs. Formally, we define the following two variations of the power domination problem.

Let $G = (V, E)$ be a graph and suppose that a given set of vertices $Z \subseteq V$ is called the *forbidden zone* of G. A set $S \subseteq V$ is called a *restricted power dominating set* of G with respect to Z (abbreviated as RPDS-Z) if S is a PDS of G such that $S \cap Z = \emptyset$. The *restricted power domination number* of G with respect to Z, denoted by $\gamma_{\hat{P}}(G, Z)$, is the minimum cardinality of an RPDS-Z of G. It is possible that Z is so restrictive that no such set S exists. In this case, we define $\gamma_{\hat{P}}(G, Z) = \infty$. An RPDS-$Z$ of G with the minimum cardinality is called a $\gamma_{\hat{P}}(G, Z)$-set, and the *restricted power domination problem* is to find a $\gamma_{\hat{P}}(G, Z)$-set. Since every RPDS-Z is a PDS, $\gamma_P(G) \leqslant \gamma_{\hat{P}}(G, Z)$. Also, it is clear that $\gamma_{\hat{P}}(G, \emptyset) = \gamma_P(G)$.

The following is another variation of the power domination problem. For a given graph $G = (V, E)$ and an integer k with $0 \leqslant k < |V|$, a set $S \subseteq V$ is called a *k-fault-tolerant power dominating set* (abbreviated as k-FPDS) of G if $S - F$ is still a PDS of G for any subset $F \subset S$ with $|F| \leqslant k$. The *k-fault-tolerant power domination number* of G, denoted by $\gamma_P^k(G)$, is the minimum cardinality of a k-FPDS of G. Similarly, a k-FPDS of G with the minimum cardinality is called a $\gamma_P^k(G)$-set, and the *k-fault-tolerant power domination problem* is to find a $\gamma_P^k(G)$-set. Obviously, $\gamma_P^k(G) \leqslant \gamma_P^{k+1}(G)$ and $\gamma_P^0(G) = \gamma_P(G)$.

Because each of the above problems can be viewed as a generalization of the power domination problem, it remains NP-complete on the aforementioned graphs. Inspired by the fact that the domination number of the grid $P_n \times P_m$ had not yet been determined for $n \geqslant 7$ and $m \geqslant n$ arbitrary, Dorfling and Henning [6] studied the power domination problem on grids and completely determined $\gamma_P(P_n \times P_m)$. In this paper, we continue this work to investigate two variations of the power domination problem on grids.

The remainder of this paper is organized as follows. Section 2 gives some auxiliary lemmas and a brief description of the work in [6]. Section 3 presents linear time algorithms to solve the restricted power domination problem on grids under the condition that consecutive rows or columns form a forbidden zone. Section 4 deals with the k-fault-tolerant power domination problem. We provide linear time algorithms to approximate a minimum placement of PMUs. In particular, we obtain performance ratios of each algorithm within a factor of 1.60, 2.34 and 3.34 for $k = 1, 2, 3$, respectively. The last section contains our concluding remarks.

2. Preliminaries

For power domination, it has been pointed out in [4,11,18] that all vertices and edges of a graph G are observed if and only if all vertices of G are observed. Thus, there is a way to simplify the problem description by using two rules instead of the original rules mentioned in Section 1. Brueni [4] first provided a simplified definition of the observation rules that requires only 2 rules. In this paper, we shall use the following equivalent definition on power domination [7,11,12]:

Observation Rule 1 (abbreviated as OR1):

A PMU on a vertex v observes v and all its neighbors.

Observation Rule 2 (abbreviated as OR2):

If an observed vertex u with degree $d > 1$ has only one unobserved neighbor v, then v becomes observed as well.

Thus, a subset $S \subseteq V$ is a PDS of G if and only if all vertices of V can be observed either by OR1 initially or by OR2 recursively. Accordingly, a PMU on a vertex may observe other vertices at an arbitrary distance when certain conditions are fulfilled by OR2 (e.g., if a PMU is placed at an endvertex of a path, then it can observe all other vertices in the path).