Discrete Applied Mathematics 157 (2009) 819–832

Contents lists available at ScienceDirect

Discrete Applied Mathematics

journal homepage: www.elsevier.com/locate/dam

Weighted coloring on planar, bipartite and split graphs: Complexity and approximation

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ARTICLE INFO

Article history:
Received 11 April 2007
Received in revised form 4 June 2008
Accepted 6 June 2008
Available online 9 July 2008

Keywords:
Graph coloring
Weighted node coloring
Weighted edge coloring
Approximability
NP-completeness
Planar graphs
Bipartite graphs
Split graphs

ABSTRACT

We study complexity and approximation of MIN WEIGHTED NODE COLORING in planar, bipartite and split graphs. We show that this problem is **NP**-hard in planar graphs, even if they are triangle-free and their maximum degree is bounded above by 4. Then, we prove that MIN WEIGHTED NODE COLORING is **NP**-hard in P_8-free bipartite graphs, but polynomial for P_5-free bipartite graphs. We next focus on approximability in general bipartite graphs and improve earlier approximation results by giving approximation ratios matching inapproximability bounds. We next deal with MIN WEIGHTED EDGE COLORING in bipartite graphs. We show that this problem remains strongly **NP**-hard, even in the case where the input graph is both cubic and planar. Furthermore, we provide an inapproximability bound of $7/6-\varepsilon$, for any $\varepsilon > 0$ and we give an approximation algorithm with the same ratio. Finally, we show that MIN WEIGHTED NODE COLORING in split graphs can be solved by a polynomial time approximation scheme.

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1. Introduction

We give in this paper some complexity results as well as some improved approximation results for MIN WEIGHTED NODE COLORING, originally studied in Guan and Zhu [15]. A k-coloring of $G = (V, E)$ is a partition $\mathcal{S} = (S_1, \ldots, S_k)$ of the node set V of G into stable sets S_i. In this case, the objective is to determine a node coloring minimizing k. A natural generalization of this problem is obtained by assigning a strictly positive integer weight $w(v) > 0$ to any node $v \in V$, and defining the weight of stable set S of G as $w(S) = \max\{w(v) : v \in S\}$. Then, the objective is to determine a node coloring $\mathcal{S} = (S_1, \ldots, S_k)$ of G minimizing the quantity $\text{val}(\mathcal{S}) = \sum_{i=1}^{k} w(S_i)$. One of the original motivations for studying this problem is related to batch scheduling. In the typical situation where jobs in a single batch are processed in parallel, the processing time of a batch equals the largest processing time of the jobs inside this batch. In the presence of pairwise incompatibilities between jobs (to be in the same batch), then minimizing the overall processing time is exactly an instance of MIN WEIGHTED NODE COLORING (where jobs are nodes, weights are processing times, and edges are incompatibilities). This problem is easily shown to be **NP**-hard; it suffices to consider $w(v) = 1, \forall v \in V$ and MIN WEIGHTED NODE COLORING becomes the classical node coloring problem. Other applications of MIN WEIGHTED NODE COLORING and other generalizations of batch coloring problems are indicated in Finke et al. [11]. Other versions of weighted colorings have been studied in Hassin and Monnot [17], Balas and Xue [1], and Frank [12].

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doi:10.1016/j.dam.2008.06.013

Consider an instance I of an **NP**-hard minimization problem Π and a polynomial time algorithm $\mathtt{A}$ computing feasible solutions for Π. Denote by $m_{\mathtt{A}}(I, S)$ the value of the Π-solution S computed by $\mathtt{A}$ on I and by $\mathrm{opt}(I)$ the value of an optimal Π-solution for I. The quality of $\mathtt{A}$ is expressed by the ratio (called approximation ratio in what follows) $\rho_{\mathtt{A}}(I) = m_{\mathtt{A}}(I, S)/\mathrm{opt}(I)$, and by the quantity $\rho_{\mathtt{A}} = \inf\{r : \rho_{\mathtt{A}}(I) < r$ for all instances I of $\Pi\}$. A very favorable situation for polynomial approximation occurs when for any $\varepsilon > 0$ there is a polynomial algorithm $\mathtt{A}_\varepsilon$ achieving a ratio bounded above by $1 + \varepsilon$. We call such algorithms $(\mathtt{A}_{\varepsilon,\varepsilon>0})$ a *polynomial time approximation scheme*, and the class of problems admitting such approximation schemes is denoted **PTAS**. Moreover, when the complexity of $\mathtt{A}_\varepsilon$ is bounded by a function $f(\varepsilon)p(|I|)$, for some polynomial p, then the scheme is called *efficient polynomial time approximation scheme*; when the complexity is bounded by a polynomial $q(1/\varepsilon, |I|)$, then it is called a *fully polynomial time approximation scheme*. The problems admitting such schemes are called respectively **EPTAS** and **FPTAS**.

Related works.

Min weighted node coloring has been introduced in [15], and further studied from a complexity and approximation viewpoint in [6]. The results in this article, extending and improving previous ones, have been originally presented at the ISAAC 2004 conference, see [9].

During the last few years, min weighted node coloring has also appeared under the name *max coloring*, in Pemmaraju et al. [25], and Pemmaraju and R. Raman [26]. In these two papers, several approximation results are given: a 4ρ-approximation is obtained for min weighted node coloring in any class of graphs for which the coloring problem admits a ρ-approximation, [26]. For instance, this implies a 4-approximation for perfect graphs. Independently from our results, an 8/7-approximation for min weighted node coloring in bipartite graphs is also obtained. Very recently, on-line versions of min weighted node coloring were presented in Epstein and Levin [10] and an off-line e-approximation is proposed for perfect graphs, [10,16]. Moreover, in Halldórsson and Shachnai [16] a polynomial algorithm with time complexity $O(n \log n)$ in paths and an efficient polynomial time approximation scheme in partial k-trees are given for min weighted node coloring, improving the time complexities given in [8] for these classes of graphs. In [25,8], the problem is shown to be **NP**-hard in interval graphs, but it is polynomially solvable by a dynamic programming algorithm in co-interval graphs [11]. A 2-approximation for min weighted node coloring in interval graphs is also obtained in [25]. The edge coloring problem has been previously studied in [22] as a special case of a non-preemptive scheduling model. In this latter paper, a greedy 2-approximation is given and an approximation within a ratio smaller than 7/6 is proved to be **NP**-hard.

Contents of the article.

We give some complexity and approximation results for min weighted node coloring. We first deal with planar graphs and we show that, for this family, the problem studied is **NP**-hard, even if we restrict our attention to triangle-free planar graphs with node-degree not exceeding 4.

We then deal with particular families of bipartite graphs. The **NP**-hardness of min weighted node coloring has been established in [6] for general bipartite graphs. We show here that this remains true even if we restrict ourselves to planar bipartite graphs or to P_{21}-free bipartite graphs, *i.e.*, bipartite graphs that do not contain induced chains on 21 vertices or more (for definitions of graph-theoretical notions used in this paper, the interested reader is referred to Berge [2]).

It is interesting to observe that these results are obtained as corollaries of a kind of generic reduction from the precoloring extension problem shown to be **NP**-complete in Bodlaender et al. [3], Hujter and Tuza [19,20], Kratochvil [21]. Then, we slightly improve the last result to P_8-free bipartite graphs and show that the problem becomes polynomial in P_5-free bipartite graphs. Observe that in [6], we have proved that min weighted node coloring is polynomial for P_4-free graphs and **NP**-hard for P_5-free graphs.

Then, we focus on approximability of min weighted node coloring in (general) bipartite graphs. As proved in [6], this problem is approximable in such graphs within an approximation ratio of 4/3; in the same paper a lower bound of $8/7-\varepsilon$, for any $\varepsilon > 0$, was also provided. Here we improve the approximation ratio of [6] by matching the 8/7-lower bound of [6] with an equal upper bound; in other words, we show here that min weighted node coloring in bipartite graphs is approximable within approximation ratio bounded above by 8/7.

We next deal with min weighted edge coloring in bipartite graphs. In this problem we consider an edge-weighted graph G and try to determine a partition of the edges of G into matchings in such a way that the sum of the weights of these matchings is minimum (analogously to the node-weighted model, the weight of a matching is the maximum of the weights of its edges). In [6], it is shown that min weighted edge coloring is **NP**-hard for cubic bipartite graphs. Here, we slightly strengthen this result by showing that this problem remains strongly **NP**-hard, in cubic and planar bipartite graphs. Furthermore, we strengthen the inapproximability bound provided in [6], by reducing it from $8/7 - \varepsilon$ to $7/6 - \varepsilon$, for any $\varepsilon > 0$. Also, we match it with an upper bound of the same value, improving so the 5/3-approximation ratio provided in [6].

Finally, we deal with the approximation of min weighted node coloring in split graphs. As proved in [6], min weighted node coloring is strongly **NP**-hard in such graphs, even if the nodes of the input graph receive only one of two distinct weights. It followed that this problem could not be solved by fully polynomial time approximation schemes, but no approximation study was addressed there. In this paper we show that min weighted node coloring in split graphs can be solved by a polynomial time approximation scheme.

In the remainder of the paper, we shall assume that for any weighted node or edge coloring $\mathcal{S} = (S_1, \dots, S_\ell)$ considered, we will have $w(S_1) \geqslant \cdots \geqslant w(S_\ell)$.

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