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Stability measure for a generalized assembly line balancing problem

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ABSTRACT

A generalized formulation for assembly line balancing problem (GALBP) is considered, where several workplaces are associated with each workstation. Thus, all tasks assigned to the same workstation have to be partitioned into blocks: each block regroups all tasks to be performed at the same workplace. The product items visit all workplaces sequentially, therefore, all blocks are proceeded in a sequential way. However, the tasks grouped into the same block are executed simultaneously. As a consequence, the execution of a block takes only the time of its longest task. This parallel execution modifies the manner to take into account the cycle time constraint. Precedence and exclusion constraints also exist for workstations and their workplaces. The objective is to assign all given tasks to workstations and workplaces while minimizing the line cost estimated as a weighted sum of the number of workstations and workplaces. The goal of this article is to propose a stability measure for feasible and optimal solutions of this problem with regard to possible variations of the processing time of certain tasks. A heuristic procedure providing a compromise between the objective function and the suggested stability measure is developed and evaluated on benchmark data sets.

1. Introduction

The design of a typical flow-oriented paced production line is considered. The line consists of a number of workstations aligned serially along a conveyor belt. Identical product items are consequently launched down the line and processed at every workstation in the order of their location. A workstation deals with only one product item at a time. The items are transferred from their current workstation to the next one at the end of each time interval called *line cycle time*. All workstations function simultaneously performing elementary tasks assigned to them. Tasks can be executed by a human operator or using special automatic machines installed at workstations.

The design aim is to partition the given set of all elementary tasks into workstations while respecting existing technological and economical constraints and optimizing one or several objectives. The set of tasks assigned to a workstation determines its load. The working time of a workstation on a product item must not be greater than the cycle time. A workstation with the greatest working time is called the *most loaded*.

This optimization problem is one of the important issues of managing assembly lines. Its simple version, the simple assembly line balancing problem or SALBP, takes into account only precedence and cycle time constraints where the sum of tasks assigned to the same workstation must be not greater than the cycle time. With regard to objectives employed, SALBPs are commonly classified into three types [25,33]: minimize the total number of opened workstations for a fixed line cycle time (SALBP-1); minimize the working time on the most loaded workstation with a fixed number of workstations (SALBP-2); and if neither the number of workstations nor line cycle time is fixed, maximize the *line efficiency* (SALBP-E). The latter objective minimizes the number of opened workstations $\times$ working time on the most loaded one. It should be emphasized that all these problems are known to be $\mathcal{NP}$-hard [26, Chapter 2.2.1.5].

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In this paper, a generalization of SALBP-1 is considered. Namely, it is supposed that each workstation is equipped with one or several workplaces (blocks) activated sequentially. At the same time, the tasks assigned to the same workplace (block) are executed simultaneously. Therefore, the working time on a workstation is determined as the sum of the working time of blocks belonging to this workstation, while the working time of a block is determined as the maximal processing time among the tasks assigned to it. The goal is to minimize the number of used equipment (i.e. the total number of workstations and workplaces). Several industrial examples of such lines can be found in [2,10,15–17] where blocks correspond to multi-spindle heads and workstations to unit-built machines.

In SALBPs, all task processing times were considered deterministic. However, these times may vary during the line lifecycle because of multiple factors, such as: operator skill, motivation and fatigue, changes in material composition of product items, product and workstation characteristics, etc. To take into account the variability of processing times, the following models are often used in the literature: *stochastic processing times* [1,3,9,12,13,23,35] and *fuzzy processing times* [14,20,34].

For stochastic models, task processing times are commonly assumed to be normally distributed independent random variables with known means and variances. In this case, chance constraints can be introduced. These constraints assure that the probability of the respect of the cycle time for each workstation will be greater than a pre-determined confidence level that is usually equal to 0.95. In fuzzy models, task processing times are represented by fuzzy intervals with given membership functions (possibility distributions) giving the grade of satisfaction of a decision maker. In that case, the assignment of tasks to workstations is implemented with respect to an introduced fuzzy arithmetic.

However, it should be noted that the application of these two models in practice is a difficult task. Indeed, available knowledge on input data is not always sufficient to deduct appropriate probability or possibility distribution functions for task processing times, especially if the design of an assembly line is planed just for one time. More often, a decision maker can only indicate a subset of tasks which processing times are subject to frequent variations. In such cases, another model can be suggested, where the set of given tasks is divided into 2 subsets of constant and variable tasks. This approach was used by Sotskov et al. [28] for SALBP-1. The authors studied the influence of variations of task processing times (VTPT) on optimal solutions constructed for completely deterministic problem. The principal goal of this approach is to determine the limit level of independent VTPT (named the *stability radius*) under which a solution remains optimal. The stability radius is an appropriate measure of credibility of known solutions in presence of VTPT. If the stability radius is known, then will be no need to reconstruct an optimal solution if the VTPT observed do not exceed it.

Note that similar approaches have been already studied for different types of combinatorial optimization problems, where along with the stability radius, another measure of sensitivity called sensitivity interval (the interval of one parameter where the solution preserves its optimality) was investigated. In what follows, we present a short review of these approaches.

Belgacem, Hifi et al. studied the sensitivity of an optimal solution for knapsack and sharing knapsack problems [4–7,18,19] subject to perturbations of profits and weights of the problem. The authors proposed algorithms for calculating the sensitivity intervals for these parameters or, as it was done in [19], while seeking an optimal solution, they adapted a branch and bound technique for calculating this interval.

In [22,36], the authors studied different aspects of sensitivity for the salesman problem. In particular, they considered the problem of seeking k best solutions under the condition that an optimal solution and its stability radius are known. A polynomial algorithm for this problem was presented for $k = 2$, and it was proved that it is $\mathcal{NP}$-hard for $k > 2$.

In [24], the authors considered the shortest path problem for the undirected graphs with m edges. They proved that the sensitivity interval for the length of an edge can be calculated in $\mathcal{O}(m + k \log k)$, where k is the number of edges of the optimal path studied.

Bräsel et al. [8], Kravchenko et al. [21], Sotskov [27] and Sotskov et al. [29–32] study the stability radius of an optimal solution in scheduling under job time uncertainty. Their works are applied on the large range of scheduling problems essentially for job shop and open shop types. They presented the necessary and sufficient conditions for the existence of the strictly positive stability radius as well as the formula of its calculation.

In this paper, we study the stability aspects for both feasible and optimal solutions for a generalized assembly line balancing problem with workplaces of parallel tasks. The remainder of the paper is organized as follows. In Section 2, basic definitions and properties are introduced. Sections 3–5 are devoted to the calculation of the stability radius for feasible, quasi-feasible (see the definition in Section 2), and optimal solutions, respectively. A heuristic procedure to find a compromise between the values of the objective function and the stability radius of a feasible solution is described in Section 6. Experimental results carried out on industrial case benchmarks are analyzed in Section 6.3. Final remarks and conclusions are given in Section 7.

2. Basic definitions and properties

2.1. Feasible, quasi-feasible and optimal solutions

All elementary tasks required to be performed constitute a given set $V = \{1, 2, \ldots, n\}$ associated with a vector $t = (t_1, t_2, \ldots, t_n) \in \mathbb{R}^n_+$ of processing times, where t_j is the processing time of task $j \in V$ and $\mathbb{R}_+$ is the set of all positive real numbers. In this paper, we consider that set V contains two types of tasks: