Osong Public Health Res Perspect 2014 5(6), 315—323
http://dx.doi.org/10.1016/j.phrp.2014.10.001
pISSN 2210-9099 eISSN 2233-6052

ORIGINAL ARTICLE

Optimal Implementation of Intervention to Control the Self-harm Epidemic

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Received: August 3, 2014
Revised: September 15, 2014
Accepted: September 22, 2014

KEYWORDS:
contagious social issue,
deliberate self-harm,
epidemiological model,
optimal control,
Pontryagin's Maximal Principle

Abstract

Objectives: Deliberate self-harm (DSH) of a young person has been a matter of growing concern to parents and policymakers. Prevention and early eradication are the main interventional techniques among which prevention through reducing peer pressure has a major role in reducing the DSH epidemic. Our aim is to develop an optimal control strategy for minimizing the DSH epidemic and to assess the efficacy of the controls.

Methods: We considered a deterministic compartmental model of the DSH epidemic and two interventional techniques as the control measures. Pontryagin's Maximum Principle was used to mathematically derive the optimal controls. We also simulated the model using the forward-backward sweep method.

Results: Simulation results showed that the controls needed to be used simultaneously to reduce DSH successfully. An optimal control strategy should be adopted, depending on implementation costs for the controls.

Conclusion: The long-term success of the optimum control depends on the implementation cost. If the cost is very high, the control could be used for a short term, even though it fails in the long run. The control strategy, most importantly, should be implemented as early as possible to attack a comparatively fewer number of addicted individuals.

1. Introduction

Deliberate self-harm (DSH) is an activity of an individual in which the sole intention is to cause self-harm, although not to commit suicide; however, sometimes acute medical situations arise [1]. More scientific definitions of DSH are available in the literature [2—4]. It is associated with the physiology and psychology of the affected individual. In the past decade, it has become a pronounced health concern among adolescents and young adults all over the world [5]. You et al [6] and Whitlock [7] addressed it as a contagious social issue. Deliberate self-harm is associated with depression, anxiety, poor school performance, family conflict [8],

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sexual abuse [9], and other factors. Mathematical modeling of epidemics is a constructive tool to assess the evolution of contagious problems and to discover strategies to reduce or eradicate the epidemic of contagious problems.

The techniques of mathematical modeling have recently been utilized in problems related to human behaviors and social interaction. For example, the theory for social behavior of individuals subjected to the social interaction was developed by Wirl and Feichtinger [10] to address the problem of obesity. Mathematical models have also been used to study the obesity epidemic [11−16]. Such ideas are also used to study smoking dynamics mathematically [17−22]. In addition, Li [23] used Bayesian proportional hazard analysis to deal with school drop-out. Porco et al [24] presented two models for antibiotic abuse. The techniques of mathematical modeling are likewise being exercised to understand contagious social and behavioral epidemics from diverse viewpoints.

Do and Lee [25] proposed a mathematical model for the self-harm epidemic and analyzed it mathematically. By considering self-harm as a contagious disease, they formulated a deterministic compartmental model. In the present study, we introduced time-dependent controls into the Do and Lee model [25], and extend an optimal control problem to understand cost-effective strategies for reducing DSH.

2. Materials and methods

2.1. Basic model

The Lee and Do model [25] without a demographic effect reduces to the following formula:

$$\begin{aligned}
\frac{dS}{dt} &= -\alpha S\frac{A+P}{N}\\
\frac{dA}{dt} &= \alpha S\frac{A+P}{N} + \omega P - \beta\frac{A}{N}(P+R) - \theta A - \eta A\\
\frac{dP}{dt} &= \beta\frac{A}{N}(P+R) + \theta A - \rho P - \omega P\\
\frac{dR}{dt} &= \eta A + \rho P
\end{aligned} \tag{1}$$

In this paper, we note that the whole population $N(t) = S(t)+A(t)+P(t)+R(t)$ is constant. The variable $N(t)$ includes only adolescents and young adults between the ages 12 years and 23 years and is divided into four classes: susceptible, $S(t)$; addicted, $A(t)$; in treatment $P(t)$; and recovered, $R(t)$. Individuals of $S(t)$ who try DSH move to $A(t)$ with the per capita transition rate, α, which is peer pressure on susceptible individuals in $A(t)$ and $P(t)$. Individuals repeating DSH remain in $A(t)$, but individuals who stop DSH move to $R(t)$ at the rate η. This is the rate at which individuals in $A(t)$ stop DSH without any treatment program or individuals who tried DSH only once and transferred to $R(t)$. When individuals in $A(t)$ seek treatment, they go to $P(t)$ at the rate of $\beta(P+R)/N+\theta$. In this equation, β is peer pressure due to individuals in $P(t)$ and $R(t)$ to the individuals in $A(t)$, and θ is the intervention rate at which addicted individuals seek treatment. If the treatment fails, individuals may go back to $A(t)$ from $P(t)$ at the rate ω. Individuals in $P(t)$ recover at rate ρ and move to $R(t)$. The values of α and β may be different, but in this study they are considered the same for homogeneous mixing. Among all of these parameters, the system is most sensitive to α and η [19]. The values of η may also increase or decrease, depending on the positive or negative influence of the Internet [26]. Furthermore, the individual who performs DSH once, seeks more serious injury for the next DSH episode [27]. Therefore, a control strategy should be concerned with prevention through controlling peer pressure α and early intervention η.

2.2. Optimal control

To shrink the DSH epidemic, we adopted two control strategies with the intent of increasing prevention [i.e., decreasing α and increasing early intervention (η)]. However, maintaining constant control over time is impractical. Therefore, our aim is to show that it is possible to implement time-dependent control techniques while minimizing the addicted population with minimum cost of implementation of the control measures.

To develop an optimal control problem for the aforementioned purpose, two control terms were introduced into the basic model (1). The model reduces to the following formula:

$$\begin{aligned}
\frac{dS}{dt} &= -\alpha(1-u_1(t))S\frac{A+P}{N}\\
\frac{dA}{dt} &= \alpha(1-u_1(t))S\frac{A+P}{N} + \omega P - \beta\frac{A}{N}(P+R)\\
&\quad -\theta A - (\eta + \mu u_2(t))A\\
\frac{dP}{dt} &= \beta\frac{A}{N}(P+R) + \theta A - \rho P - \omega P\\
\frac{dR}{dt} &= (\eta + \mu u_2(t))A + \rho P
\end{aligned} \tag{2}$$

In this equation $N = S(t)+A(t)+P(t)+R(t)$ is constant.

The control variables, $u_1(t)$ and $u_2(t)$, represent the quantity of intervention associated with the parameters α and η, respectively at time t. The factor of $1-u_1(t)$ reduces the per capita transition rate α from $S(t)$ to $A(t)$. The per capita transition rate η from P to R increases at a rate that is proportional to $u_2(t)$ in which $\mu > 0$ is the proportionality constant.

We define our control set as follows: