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A greedy approximation algorithm for the group Steiner problem

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Abstract

In the group Steiner problem we are given an edge-weighted graph $G=(V,E,w)$ and m subsets of vertices $\{g_i\}_{i=1}^{m}$. Each subset g_i is called a *group* and the vertices in $\bigcup_i g_i$ are called *terminals*. It is required to find a minimum weight tree that contains at least one terminal from every group.

We present a poly-logarithmic ratio approximation for this problem when the input graph is a tree. Our algorithm is a recursive greedy algorithm adapted from the greedy algorithm for the directed Steiner tree problem [Approximating the weight of shallow Steiner trees, Discrete Appl. Math. 93 (1999) 265–285, Approximation algorithms for directed Steiner problems, J. Algorithms 33 (1999) 73–91]. This is in contrast to earlier algorithms that are based on rounding a linear programming based relaxation for the problem [A polylogarithmic approximation algorithm for the Group Steiner tree problem, J. Algorithms 37 (2000) 66–84, preliminary version in Proceedings of SODA, 1998 pp. 253–259, On directed Steiner trees, Proceedings of SODA, 2002, pp. 59–63]. We answer in positive a question posed in [A polylogarithmic approximation algorithm for the Group Steiner tree problem, J. Algorithms 37 (2000) 66–84, preliminary version in Proceedings of SODA, 1998 pp. 253–259] on whether there exist good approximation algorithms for the group Steiner problem that are not based on rounding linear programs. For every fixed constant $\varepsilon>0$, our algorithm gives an $O((\log \sum_i |g_i|)^{1+\varepsilon} \cdot \log m)$ approximation in polynomial time. Approximation algorithms for trees can be extended to arbitrary undirected graphs by probabilistically approximating the graph by a tree.

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This results in an additional multiplicative factor of $O(\log |V|)$ in the approximation ratio, where $|V|$ is the number of vertices in the graph. The approximation ratio of our algorithm on trees is slightly worse than the ratio of $O(\log(\max_i |g_i|) \cdot \log m)$ provided by the LP based approaches.

Keywords: Group Steiner problem; Tree; Combinatorial; Greedy; Approximation algorithm

1. Introduction

The Steiner tree problem is among the fundamental problems in network design. The input to the Steiner tree problem is an undirected edge-weighted graph $G = (V, E, w)$ and a set of terminals $\mathcal{T} \subseteq V$. The objective is to find a minimum weight tree T that spans the terminals in $\mathcal{T}$. The Steiner tree problem is known to be NP-hard [13] and also APX-hard [5]. In this paper we consider the group Steiner problem which is a generalization of the Steiner tree problem. The input to this problem also consists of an edge-weighted graph $G = (V, E, w)$; however, instead of a single set of terminals we are given a collection of possibly intersecting subsets of vertices $\{g_i\}_i$. Each subset g_i is called a group. The objective is to find a minimum weight tree that contains at least one vertex from each group. Throughout, we denote the number of groups by m, the number of terminals $|\cup_{i=1}^{m} g_i|$ by n, and the size of the largest group $\max_i |g_i|$ by N. We note that the sum of the group sizes $\sum_{i=1}^{m} |g_i|$ is at most mn.

The group Steiner problem was introduced by Reich and Widmayer [23] motivated by applications to wire routing with multi-port terminals in physical VLSI design. See [14] for additional references to this problem. The problem is of interest not only because of its applications but also because of its relation to the Steiner tree problem in both undirected and directed graphs. The search for good approximation algorithms for this problem has inspired new technical ideas [14,19,20,25].

The group Steiner problem is a strict generalization of the Steiner tree problem, and in [14] it is shown that very special cases of the group Steiner problem are harder to approximate than the Steiner tree problem: in particular it is shown that the set cover problem can be reduced in an approximation preserving way to the group Steiner problem on star graphs. From the hardness of approximating set cover [12,22], it follows that the group Steiner problem on stars, and hence trees, is NP-hard to approximate to within a factor better than $c \ln m$ for some constant c, or to a factor better than $(1 - o(1)) \ln m$ unless $NP \subseteq DTIME(n^{\log \log n})$. In recent work, Halperin and Krauthgamer [16] improved the hardness of approximation. They showed that for every $\varepsilon > 0$, the group Steiner problem on trees is hard to approximate to within a factor better than $\Omega(\log^{2-\varepsilon} m)$, unless NP problems can be solved by quasi-polynomial time Las-Vegas algorithms.

In terms of upper bounds, the first sub-linear approximation ratio for this problem was an $O(\sqrt{m})$ ratio given by Bateman et al. [4]. Garg et al. [14] improved this substantially and obtained the first poly-logarithmic approximation ratio for this problem. They gave an $O(\log N \log m)$ approximation algorithm for the problem on *trees* based on an elegant randomized rounding of the natural linear programming relaxation for the problem. It should be mentioned that their algorithm achieves a ratio of $O(\min\{h, \log N\} \log m)$ on