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On minimizing the number of ADMs in a general topology optical network

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ABSTRACT

Minimizing the number of electronic switches in optical networks has been a main research topic in some recent studies. In such networks, we assign colors to a given set of lightpaths, and they are partitioned into unicolor cycles and paths; the switching cost is minimized when the number of paths is minimized. Most approximation and heuristic algorithms for this problem have a preprocessing stage, in which possible cycles are found. Among them, the basic algorithm eliminates cycles of size at most l, and has a performance guarantee of $OPT+\frac{1}{2}(1+\epsilon)N$, where OPT is the cost of an optimal solution, N is the number of lightpaths and $0 \le \epsilon \le \frac{1}{l+2}$, for any given odd l. The time complexity of the algorithm is exponential in l. We improve the analysis of this algorithm, by showing that $\epsilon \le \frac{1}{\frac{3}{2}(l+2)}$, which implies a reduction of the exponent in the time complexity. We also improve the lower bound by showing that $\epsilon \ge \frac{1}{2l+3}$. The results shed more light on the structure of this basic algorithm. In addition, in our analysis we suggest a novel technique – including a new combinatorial lemma – to deal with this problem.

1. Introduction

1.1. Background

Given a WDM network $G = (V, E)$, comprising optical nodes and a set of full-duplex lightpaths $P = \{p_1, p_2, \dots, p_N\}$ of G, the wavelength assignment (WLA) task is to assign a wavelength to each lightpath p_i.

In the following discussion, we also assume that each lightpath $p \in P$ is contained in a cycle of G. Each lightpath p uses two ADM's, one at each endpoint. Although only the downstream ADM function is needed at one end, and only the upstream ADM function is needed at the other end, full ADM's will be installed on both nodes in order to complete the protection path around some ring. The full configuration would result in a number of SONET rings. It follows that if two adjacent lightpaths are assigned the same wavelength, then they can be used by the same SONET ring and the ADM in the common node can be shared by them. This would save the cost of one ADM. An ADM may be shared by at most two lightpaths. A more detailed technical explanation can be found in [5].

Lightpaths sharing ADM's in a common endpoint can be thought as concatenated, so that they form longer paths or cycles. Each of these longer paths/cycles does not use any edge $e \in E$ twice, for, otherwise they cannot use the same wavelength and this is a necessary condition to share ADM's.

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1.2. Previous work

Minimizing the number of electronic switches in optical networks has been a main research topic in some recent studies. The problem was introduced in [5] for ring topology. An approximation algorithm for ring topology, with approximation ratio of 3/2 was presented in [2], and was improved in [6,3] to $10/7+\epsilon$ and 10/7, respectively.

For general topology [4] describe an algorithm with an approximation ratio of 8/5. The same problem was studied in [1] and an algorithm was presented that has a preprocessing phase, where cycles of length at most l are included in the solution; this algorithm was shown to have performance guarantee of

$$OPT + \frac{1}{2}(1+\epsilon)N, \quad 0 \le \epsilon \le \frac{1}{l+2} \tag{1}$$

where OPT is the cost of an optimal solution, N is the number of lightpaths, for any given odd l. The dominant part in the running time of the algorithm is the preprocessing phase, that is exponential in l.

1.3. Our contribution

We improve the analysis of the algorithm of [1] and prove a performance of

$$OPT + \frac{1}{2}(1+\epsilon)N, \qquad \frac{1}{2l+3} \le \epsilon \le \frac{1}{\frac{3}{2}(l+2)}. \tag{2}$$

Specifically, we show that the algorithm guarantees to satisfy an upper bound of $OPT + \frac{1}{2}(1+\epsilon)N$, for $\epsilon \le \frac{1}{\frac{3}{2}(l+2)}$, and we demonstrate a family of instances for which the performance of the algorithm is $OPT + \frac{1}{2}(1+\epsilon)N$, for $\epsilon \ge \frac{1}{2l+3}$.

Our analysis sheds more light on the structure and properties of the algorithm, by closely examining the structural relation between the solution found by the algorithm vs an optimal solution, for any given instance of the problem.

As the running time of the algorithm is exponential in l, our result implies an improvement in the analysis of the running time of the algorithm. For any given $\epsilon > 0$, the exponent of the running time needed to guarantee the approximation ratio $(3+\epsilon)/2$ is reduced by a factor of 3/2.

In addition, in the development of our bounds we use a purely combinatorial problem, that is of interest by itself.

In Section 2 we describe the problem and some preliminary results. The algorithm and its analysis are presented in Section 3. We conclude with discussion and open problems in Section 4.

2. Problem definition and preliminary results

2.1. Problem definition

An instance α of the problem is a pair $\alpha = (G, P)$ where $G = (V, E)$ is an undirected graph and P is a set of simple paths in G. Given such an instance we define the following:

Definition 2.1. The paths $p, p' \in P$ are conflicting or overlapping if they have an edge in common. This is denoted as $p \asymp p'$. The graph of the relation $\asymp$ is called the conflict graph of (G, P).

Definition 2.2. A proper coloring (or wavelength assignment) of P is a function $w : P \mapsto \mathbb{N}$, such that $w(p) \neq w(p')$ whenever $p \asymp p'$.

Note that w is a proper coloring if and only if for any color $c \in \mathbb{N}$, $w^{-1}(c)$ is an independent set in the conflict graph.

Definition 2.3. A valid chain (resp. cycle) is a path (resp.cycle) formed by the concatenation of distinct paths $p_0, p_1, \ldots, p_{k-1} \in P$ that do not go over the same edge twice. Note that the paths of a valid chain (resp. cycle) constitute an independent set of the conflict graph.

Definition 2.4. A solution S of an instance $\alpha = (G, P)$ is a set of chains and cycles of P such that each $p \in P$ appears in exactly one of these sets.

In the following, we introduce the shareability graph, that together with the conflict graph constitutes another (dual) representation of the instance α. In the following, except one exception, we will use the dual representation of the problem.

Definition 2.5. The shareability graph of an instance $\alpha = (G, P)$, is the edge-labeled multi-graph $\mathcal{G}_\alpha = (P, E_\alpha)$ such that there is an edge $e = (p, q)$ labeled u in E_α if and only if $p \not\asymp q$, and u is a common endpoint of p and q in G.