Discrete Applied Mathematics 157 (2009) 402–405

Contents lists available at ScienceDirect

Discrete Applied Mathematics

journal homepage: www.elsevier.com/locate/dam

Note

On a new code, $[2^n - 1, n, 2^{n-1}]$

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ARTICLE INFO

Article history:
Received 18 February 2008
Received in revised form 17 June 2008
Accepted 18 June 2008
Available online 5 August 2008

Keywords:
Linear code
Support
Design
Parity sectioned reduction
Sectioned code
Non-uniform error correction

ABSTRACT

A binary linear code in F_2^n with dimension k and minimum distance d is called an $[n, k, d]$ code. A t-(n, m, λ) design D is a set X of n points together with a collection of m-subsets of X (called a block) such that every t-subset of X is contained in exactly λ blocks. A constant length code which corrects different numbers of errors in different code words is called a non-uniform error correcting code. Parity sectioned reduction of a linear code gives a non-uniform error correcting code. In this paper a new code, $[2^n - 1, n, 2^{n-1}]$, is developed. The error correcting capability of this code is $2^{n-2} - 1 = e$. It is shown that this code holds a 2-$(2^n - 1, 2^{n-1}, 2^{n-2})$ design. Also the parity sectioned reduction code after deleting the same g $(\leq e)$ positions of each code word of this code holds a 1-$(2^n - 1 - g, 2^{n-1} - j, {}^gC_j .2^{n-1-g})$ design for $n \geq 3$, $g = 1, 2, \ldots, e$ and $j = 0, 1, \ldots, g$.

1. Introduction

It is important to define a new code that can be encoded or decoded efficiently with error correcting ability.

A generator matrix for the $[n, k, d]$ linear code C over F_2^n is a $k \times n$ matrix G whose rows are linearly independent of $C = \mathbf{RS}(G)$, the row space of G.

In this paper, the systematic generator matrix for a new code, $[2^n - 1, n, 2^{n-1}]$, is defined. The different properties of this code are stated and proved.

Let P_n be a matrix of order $2^{n-1} \times n$. The rows of P_n are all binary code words of length n except the $\bar{0}$ code word.

In this paper, a systematic generator matrix of the new code is designed via $G_n = [P_n]^t$. Now we consider the square matrix C_n^* of order $2^n - 1$ whose rows are all the code words generated by G_n except the $\bar{0}$ code word.

For example,

$$P_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \quad G_3 = \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{pmatrix}, \quad C_3^* = \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

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0166-218X/$ – see front matter
doi:10.1016/j.dam.2008.06.029

$$P_4 = \begin{pmatrix} 1&0&0&0\\ 0&1&0&0\\ 0&0&1&0\\ 0&0&0&1\\ 1&1&0&0\\ 1&0&1&0\\ 1&0&0&1\\ 0&1&1&0\\ 0&1&0&1\\ 0&0&1&1\\ 1&1&1&0\\ 1&1&0&1\\ 1&0&1&1\\ 0&1&1&1\\ 1&1&1&1 \end{pmatrix}, \qquad G_4 = \begin{pmatrix} 1&0&0&0&1&1&1&0&0&0&1&1&1&0&1\\ 0&1&0&0&1&0&0&1&1&0&1&1&0&1&1\\ 0&0&1&0&0&1&0&1&0&1&1&0&1&1&1\\ 0&0&0&1&0&0&1&0&1&1&0&1&1&1&1 \end{pmatrix}$$

$$C_4^* = \begin{pmatrix} 1&0&0&0&1&1&1&0&0&0&1&1&1&0&1\\ 0&1&0&0&1&0&0&1&1&0&1&1&0&1&1\\ 0&0&1&0&0&1&0&1&0&1&1&0&1&1&1\\ 0&0&0&1&0&0&1&0&1&1&0&1&1&1&1\\ 1&1&0&0&0&1&1&1&1&0&0&0&1&1&0\\ 1&0&1&0&1&0&1&1&0&1&0&1&0&1&0\\ 1&0&0&1&1&1&0&0&1&1&1&0&0&1&0\\ 0&1&1&0&1&1&0&0&1&1&0&1&1&0&0\\ 0&1&0&1&1&0&1&1&0&1&1&0&1&0&0\\ 0&0&1&1&0&1&1&1&1&0&1&1&0&0&0\\ 1&1&1&0&0&0&1&0&1&1&1&0&0&0&1\\ 1&1&0&1&0&1&0&1&0&1&0&1&0&0&1\\ 1&0&1&1&1&0&0&1&1&0&0&0&1&0&1\\ 0&1&1&1&1&1&1&0&0&0&0&0&0&1&1\\ 1&1&1&1&0&0&0&0&0&0&1&1&1&1&0 \end{pmatrix}$$

and so on.

2. Definitions

1. *Support* [3]: Let $\bar{x}$ be a binary word of length n. The set of positions in which $\bar{x}$ has non-zero entries is called the support of $\bar{x}$.

2. *Design* [4]: Let C be a binary code of length n. Let S_w be the set of code words in C of weight w. We say that S_w holds a t-(n, w, λ) design if the supports of code words in S_w form the blocks of a t-(n, w, λ) design, and if for any t-set $T \subset \{1, 2, \ldots, n\}$ there are exactly λ code words of weight w in C with 1's in the positions given by T.

3. *Parity sectioned reduction* [1]: Let C be a binary e-error correcting (n, k) linear systematic code with parity check matrix $H_{n-k,n} = [A|I_{n-k}]$ and error range inequalities

$$\sum_{j=1}^{n} |x_j - c_{i,j}| \le e, \quad i = 1, 2, \ldots, 2^k.$$

By g-parity sectioned reduction of the code C, we mean the following operations on the parity check matrix $H_{n-k,n}$ and the error range inequalities:

1. Select some g $(\le e)$ parity check positions for sectioning; if the code is sectioned at the pth check position, then delete the pth column and row of I_{n-k}. A reduced matrix $H_{n-k-g,n-g} = [A' : I_{n-k-g}]$ is obtained.
2. In each code word of C, delete the g-parity check digits; in the error range inequalities, assign values from (0, 1) to the variables corresponding to these g positions.

3. Properties

Property 1. *The matrix C_n^* can be rearranged in a manner such that the transpose of this matrix is equal to itself.*

Property 2. *The Hamming weight (i.e. support) of each code word of C_n is 2^{n-1}.*

Property 3. *The code C_n is self-orthogonal for $n > 2$.*