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On the characterization of the domination of a diameter-constrained network reliability model

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Abstract

Let $G=(V,E)$ be a digraph with a distinguished set of terminal vertices $K \subseteq V$ and a vertex $s \in K$. We define the s, K-diameter of G as the maximum distance between s and any of the vertices of K. If the arcs fail randomly and independently with known probabilities (vertices are always operational), the diameter-constrained s, K-terminal reliability of G, $R_{s,K}(G, D)$, is defined as the probability that surviving arcs span a subgraph whose s, K-diameter does not exceed D.

The diameter-constrained network reliability is a special case of coherent system models, where the domination invariant has played an important role, both theoretically and for developing algorithms for reliability computation. In this work, we completely characterize the domination of diameter-constrained network models, giving a simple rule for computing its value: if the digraph either has an irrelevant arc, includes a directed cycle or includes a dipath from s to a node in K longer than D, its domination is 0; otherwise, its domination is -1 to the power $|E|-|V|+1$. In particular this characterization yields the classical source-to-K-terminal reliability domination obtained by Satyanarayana.

Based on these theoretical results, we present an algorithm for computing the reliability.

Keywords: Graph theory; Domination; Diameter; Network reliability

1. Introduction and reliability model

In this paper we are concerned with digraphs (directed digraphs) $G=(V,E)$, where V and E are the set of vertices and arcs of G, respectively. A subgraph $G'=(V',E')$ of a digraph $G=(V,E)$ is a digraph such that $V' \subseteq V$, and $E \subseteq E$, and G' is a spanning subgraph of G if $V'=V$.

For a digraph $G=(V,E)$, we denote a dipath P from vertex u to vertex v (also called a u, v-dipath) in G as $P=\langle (u_1=u, u_2), (u_2, u_3), \ldots, (u_{r-1}, u_r=v)\rangle$, where the vertices of P are distinct, and (u_j, u_{j+1}) is an arc of G. Moreover, let $r-1$ be the length of P. A directed cycle (dicycle) C in G is obtained from P by allowing $u=v$. Furthermore, we say that a digraph G is cyclic if it contains a directed cycle; otherwise G is acyclic.

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The distance between vertex u and vertex v of a digraph G is defined as the length of the shortest dipath from vertex u to vertex v in G, and in the case that no dipath exists between u and v, the distance is infinite.

A communication network can be modeled by a digraph (also called its underlying graph) G, where the set of nodes (e.g., packet switches) and communication links of the network are the vertices and arcs of G, respectively. Moreover, since a communication network could be subject to random failures of its components, we represent this probabilistic behavior of the network by assigning failure probabilities to the vertices and/or arcs of its underlying digraph. A widely used probabilistic model is the one where arcs fail randomly and independently with known probabilities, and where vertices are always operational; when we mention a probabilistic digraph, we will refer to this model.

Let $G=(V,E)$ be a probabilistic digraph, with terminal vertex set $K \subseteq V$, and distinguished vertex $s \in K$ (called the source node), and diameter bound D. The s,K-diameter of G is defined as the maximum distance from s to any other vertex $u \in K$. By the definition of distance, in the case that no dipath exists from vertex s to some vertex $u \in K$, then the s,K-diameter is infinite.

The diameter-constrained s,K-terminal reliability $R_{s,K}(G,D)$ is defined [13] as the probability that the surviving arcs span a subgraph of G whose s,K-diameter does not exceed D, or equivalently, as the probability that for each vertex $u \in K$, there exists an operating dipath (i.e., a dipath composed of surviving arcs of G) from s to u of at most D arcs. This reliability measure subsumes the classical source-to-K-terminal reliability $R_{s,K}(G)$ of a probabilistic digraph G (see [18] for a complete discussion on this subject), which is the probability that the surviving arcs span a subgraph where there exists an operational dipath between s and u, $u \in K$: noting that the longest dipath in G has at most $n-1$ arcs, where n is the number of nodes of G, we have that $R_{s,K}(G)$ is equal to $R_{s,K}(G,D)$ for $D=n-1$.

As the classical reliability does not take into account the length of the dipaths connecting the terminal nodes of a digraph G, this reliability model was extended to assess the probability that there are short-enough dipaths from the source vertex s to a set of terminal vertices of G. That is the case in multicasting-routing with end-to-end delay constraints, where a source node must broadcast messages to a set of destination nodes in a network (e.g., teleconference), while these messages must meet certain delay constraints. This problem can be modeled as a digraph with a source node s, a set M of destination nodes, and where each arc is assigned a weight corresponding to the delay to be experienced by a packet traveling along this arc.

A line of research in this area is the study of techniques to obtain diameter-constrained Steiner trees, in order to ensure that a packet traveling from the source to a terminal node can arrive within the allowed delays [5–7,11,15]. Another area of research consists in meeting with diameter and two-connectivity objectives by extending an existing topology with new arcs [4]. To our knowledge none of these previous works take into account the operational probability of the network components, thus the diameter-constrained reliability measure may be applied to determine the suitability of a network to meet end-to-end delay constraints.

The domination of a digraph is a graph-theoretic measure which appeared in alternative formulations for improving the efficiency of evaluating the classical reliability. In this paper, we give a characterization of the domination of a digraph, for the diameter-constrained reliability model, which in particular yields the domination for the classical case. For a digraph $G=(V,E)$, with terminal set K, source node $s \in K$, and diameter bound D, the diameter-constrained s,K-terminal reliability can be computed as a sum of terms involving the domination of some spanning subgraphs of G. Even though, for general graphs, the computation of the reliability remains a $\#P$-complete problem, since the evaluation of the classical source-to-K-terminal reliability belongs to this computational class (see [14]), the application of the domination for calculating the diameter-constrained reliability substantially reduces the computational effort.

In Section 2, we present some preliminary definitions and notation that will be used in the following sections, and introduce the domination for general systems. In Section 3 we give a complete characterization of the diameter-constrained reliability domination of a digraph. In Section 4 we formally prove this domination characterization, and finally, in Section 5, we present an algorithm based on the previous theoretical results.

The notation in this paper follows that of Harary [8], unless otherwise noted.

2. Preliminaries and domination

As we are considering digraphs, we use the notation $ind_G(u)$ and $outd_G(u)$ to denote the indegree and outdegree of vertex u in G, respectively, where the indegree of u is the number of arcs directed into u, while the outdegree is the number of arcs emanating from u.