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# The Capacitated Orienteering Problem

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ABSTRACT

In the Orienteering Problem, we are given an undirected, metric graph $G = (V, E)$, starting and end nodes $s, t \in V$, node profits $\pi : V \to \mathbb{N}$ and a length bound $D$. The goal is to find an $s$–$t$ path of length at most $D$ that collects maximum profit. The Orienteering Problem is a fundamental network design problem and efficient algorithms for this problem have often been used as subroutine to develop efficient algorithms for a wide number of vehicle routing problems.

The focus of this paper is on a natural generalization in which we also consider node demands $r : V \to \mathbb{N}$ and a capacity bound $C$. The goal is to find an $s$–$t$ path of length at most $D$ that collects maximum profit from nodes whose total demand does not exceed the capacity bound $C$.

We give a $(3 + \varepsilon)$-approximation algorithm for the Capacitated Orienteering Problem in general graphs, which improves over the previously best known approximation bound. We further obtain a PTAS on trees and a PTAS on Euclidean metrics.

As one may expect, there is a number of capacitated vehicle routing problems where the Capacitated Orienteering Problem appears as subroutine. As a byproduct of our analysis, we develop new approximation results for some of those problems.



## 1. Introduction

In the Orienteering Problem (OP), we are given an undirected complete graph $G = (V, E)$ with metric distances $d : E \to \mathbb{N}$ on the edges, starting and end nodes $s, t \in V$, node profits $\pi : V \to \mathbb{N}$ and a length bound $D$. The goal is to find an $s$–$t$ path $P = (V_P, E_P)$, subject to the length constraint $\sum_{e \in E_P} d(e) \leq D$, that maximizes the profit $\sum_{v \in V_P} \pi(v)$ collected by the visited nodes.

OP is a fundamental network design problem with high theoretical and practical relevance. The problem is widely studied in the literature for approximation, exact, and heuristic algorithms [12–14,24]. Indeed, there is a long list of vehicle routing problems, for which efficient algorithms rely on applying algorithms for OP as a subroutine (e.g. Deadline-TSP [6], Distance-constrained Vehicle Routing Problem [22], School Bus Problem [8]).

In this paper, we focus on a natural generalization of OP, in which the selected path has to satisfy a second budget constraint besides the length bound. Given node demands $r : V \to \mathbb{N}$ and a capacity bound $C \in \mathbb{N}$, we look for a path $P$ that maximizes $\sum_{v \in V_P} \pi(v)$ and that satisfies both constraints $\sum_{e \in E_P} d(e) \leq D$ and $\sum_{v \in V_P} r(v) \leq C$. We refer to this problem as the *Capacitated* Orienteering Problem (COP).

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Gupta et al. [18] have recently shown that there is a constant factor approximation algorithm for COP. In particular, they design a $2\alpha$-approximation algorithm for COP, relying on an $\alpha$-approximation algorithm for OP as a subroutine. Using the currently best known approximation algorithm for OP given by Chekuri et al. [12] with factor $(2+\varepsilon)$, their result translates into a $(4+\varepsilon)$-approximation algorithm for COP.

In this paper, we give a $(1+\alpha+\varepsilon)$-approximation algorithm for COP, relying on an $\alpha$-approximation algorithm for OP as a subroutine. Using the result of [12], this implies an approximation guarantee of $(3+\varepsilon)$, improving over [18].

We also give a Polynomial Time Approximation Scheme (PTAS) for COP on tree metrics and on Euclidean metrics. Both results rely on dynamic programming combined with a *feasibilization* method introduced by Grandoni et al. [17]. The PTAS for COP on Euclidean metrics builds upon the corresponding known result for OP by Chen and Har-Peled [13].

Additionally, we observe that for some capacitated vehicle routing problems, good approximation algorithms can be built upon the existence of good approximation algorithms for COP.

A first extension considers a fleet of $M$ capacitated vehicles that can be used to collect profits. This is the so-called Capacitated Team Orienteering Problem (CTOP) that is motivated by several applications in logistics [3]. As an example, consider a carrier that has to select the most profitable customers whose demands can be served by his capacitated fleet of vehicles.

A variation of this problem is the Capacitated Team Orienteering with Split Deliveries (SDCTOP) [2], which is defined as CTOP but with the difference that a customer's demand can be served by more than one vehicle (split delivery). Motivation for studying this problem comes from the fact that in some applications important savings can be achieved if a customer can be served by more than one vehicle (see [5]). Both CTOP and SDCTOP have been studied intensively in terms of heuristics and exact algorithms, but there is no rigorous analysis of the complexity of these problems in terms of approximation algorithms. We give a constant factor approximation algorithm for both problems, using our approximation results for COP.

Another application concerns the Distance-constrained Vehicle Routing Problem (DVRP). This problem asks for the minimum number of paths respecting both a length bound $D$ and a capacity bound $C$ that is necessary to serve all customer demands. Nagarajan and Ravi [22] obtained a $\min\{O(\log n), O(\log D)\}$-approximation algorithm, where $n$ denotes the number of customers. Recently, a $O\left(\frac{\log D}{\log\log D}\right)$-approximation algorithm was presented by Friggstad and Swamy [15]. Combining our results with the greedy algorithm for the set cover problem, we easily obtain an $O(\log C)$ approximation.

### 1.1. Related work

OP was introduced by Golden et al. [16] and is NP-hard already on star graphs. However, it admits a $(2+\varepsilon)$-approximation algorithm for general metrics [12] and a PTAS for Euclidean metrics [13].[1] Gupta, Krishnaswamy, Nagarajan and Ravi [18] give approximation algorithms for a stochastic version of OP. Among their results, they obtain a $(4+\varepsilon)$-approximation algorithm for COP (which they call Knapsack Orienteering).

A survey on routing problems with profits is due to Feillet, Dejax, Gendreau [14]. Recently, Vansteenwegen, Souffriau and Van Oudheusden [24] gave a survey on heuristics for OP and related problems. Butt and Cavalier [10] introduced the Multiple Tour Maximum Collection Problem which is the natural extension to a fleet of vehicles. This problem was first referred to as Team Orienteering Problem by Chao, Golden and Wasil [11]. Recent heuristics for the Team Orienteering Problem were presented by Tang, Miller-Hooks [23] and by Archetti, Hertz and Speranza [4], whereas an exact algorithm was presented by Boussier, Feillet, Gendreau [9].

The CTOP was first studied by Archetti et al. [3] who described heuristics and a branch-and-price algorithm. The algorithms were extended to SDCTOP by Archetti et al. [2]. They also showed that by allowing split deliveries, the profit of the optimal solution can be at most twice as large as the optimal profit for CTOP.

## 2. Constant approximation algorithm for general graphs

Recall that an $\alpha$-approximation algorithm for a maximization problem is a polynomial time algorithm that returns, for any instance of the problem, a feasible solution whose objective value is at least $\frac{1}{\alpha}$ times the value of an optimal solution.

In this section, we present a $(3+\varepsilon)$-approximation algorithm for COP. We let $\alpha_{op}$ be the best approximation factor known for the OP. The result follows from Theorem 1 and the $(2+\varepsilon)$-approximation algorithm for OP due to [12].

In the following, we use the notation $r(W) := \sum_{v\in W} r(v)$ and analogously $\pi(W) := \sum_{v\in W} \pi(v)$ for any subset of nodes $W$. For a given path $P$, we denote by $V_P$ the nodes of $P$ and by $E_P$ its edges. With a slight abuse of notation, we shorten $r(V_P)$ and $\pi(V_P)$ to $r(P)$ and $\pi(P)$ respectively. Without loss of generality,[2] we assume in the following that $\frac{\pi(v)}{r(v)} \geq 1$ for all nodes $v$.

1 In fact, the result in [13] is given for unit profit values but it can be generalized to arbitrary profits by simply scaling the profits (cf. [7]).

2 This can be achieved e.g. by removing from $G$ nodes with zero profit (if any) and by multiplying all profits by the product of all non zero demand values.