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Diagnosis of constant faults in read-once contact networks over finite bases

Monther Busbait, Igor Chikalov, Shahid Hussain, Mikhail Moshkov *

Computer, Electrical and Mathematical Sciences and Engineering Division, King Abdullah University of Science and Technology, Thuwal 23955-6900, Saudi Arabia

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ABSTRACT

We study the depth of decision trees for diagnosis of constant 0 and 1 faults in read-once contact networks over finite bases containing only indecomposable networks. For each basis, we obtain a linear upper bound on the minimum depth of decision trees depending on the number of edges in the networks. For bases containing networks with at most 10 edges we find coefficients for linear bounds which are close to sharp.

1. Introduction

In this paper, we consider the representation of Boolean functions by contact networks (networks). These are undirected graphs containing two poles (designated nodes), with a Boolean variable assigned to each edge. For a given tuple of variable values, the value of the function corresponding to the network is equal to 1 if and only if there exists a path between poles such that the value of each variable assigned to the edges in this path is equal to 1. The contact networks are also known as contact schemes, contact circuits, and switching circuits—see papers by Shannon [18], Chegis and Yablonskii [4], and Karpova [9]. We consider only read-once contact networks (also known as iteration-free, non-repeating, and without repetition) in which pairwise different variables are assigned to the edges.

We study read-once networks over a finite set B of networks. These networks can be obtained by replacing some edges in a network from B with recursively constructed networks (including networks from B). We assume that all networks from B are indecomposable, i.e., cannot be obtained from two nontrivial (containing more than one edge) networks by replacing an edge in the first network with the second one. Read-once networks over a finite set B of networks are very closely related to read-once formulas over the set of Boolean functions corresponding to the networks from B.

We consider constant faults of read-once networks, each of which consists in assigning constant values 0 and 1 to some of the network variables. We study the problem of diagnosis of constant faults: for a given faulty network we should recognize the function implemented by this network. To solve this problem we use decision trees with membership queries: we can ask about the value of the function implemented by a given faulty network on an arbitrary tuple of values of the network variables. The depth of a decision tree is the maximum number of nonterminal nodes (queries) in a path from the root to a terminal node. For a read-once network S over B, we denote by $L(S)$ the number of edges in S and by $h(S)$—the minimum depth of a decision tree for diagnosis of S.

* Corresponding author. Tel.: +966 02 808 0334; fax: +966 02 802 1291.
E-mail address: mikhail.moshkov@kaust.edu.sa (M. Moshkov).

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We prove that $h(S) \leq t(B)(L(S) - 1)$ for any read-once network S over B, where $t(B) = \max\{t(Q) : Q \in B\}$ and $t(Q) = \frac{h(Q)}{L(Q)-1}$ for any $Q \in B$. This result was presented by Moshkov in [12] without proof. We also consider the function $H_B(n)$ which characterizes the dependence in the worst case of the minimum depth of a decision tree for diagnosis of a network over B on the number n of edges in the network, and show that $H_B(n) = \Theta(n)$.

Kuznetsov in [10] published the list of all nontrivial indecomposable networks with at most 10 edges. We obtain upper and lower bounds for the parameters $h(Q)$ and $t(Q)$ for each indecomposable network Q with at most 10 edges. The difference between lower and upper bounds on the parameter $h(Q)$ is at most one. Goduhina in [6] obtained lower and upper bounds on the parameters $h(Q)$ and $t(Q)$ for each indecomposable network Q with at most eight edges. For two out of ten networks, we improved the bounds obtained by Goduhina.

The obtained results give, for an arbitrary basis B containing only indecomposable networks with at most 10 edges, an upper bound $h(S) \leq t(B)(L(S) - 1)$ for networks S over B if we know value $t(B)$ or an upper bound $h(S) \leq t^u(B)(L(S) - 1)$ if we know an upper bound $t^u(B)$ on $t(B)$.

In some cases, the universal bound $h(S) \leq t(B)(L(S) - 1)$ can be improved for decomposable networks S based on the thorough study of read-once networks over B. As an example, let us consider the basis $B(2) = \{S_1^2, S_2^2\}$ (see Table 1 for corresponding figures). The network S_1^2 consists of two poles and two parallel edges between the poles, and implements the function $x \vee y$. The network S_2^2 consists of two poles and two edges that form a path of the length two between the poles, and implements the function $x \wedge y$. We prove that $t(B(2)) = 3$. Therefore, $h(S) \leq 3(L(S) - 1)$ for any read-once network over $B(2)$. From the results obtained by Madatyan in [11], it follows that $h(S) \leq \frac{3}{2}L(S)$ for any read-once network over $B(2)$.

We should mention three directions of research which are connected with the topic of our paper.

The first direction is related to the study of diagnostic tests for constant faults in read-once networks. A diagnostic test is a set of (non-adaptive) membership queries which allows us to solve the problem of diagnosis for constant faults. The most interesting results here were obtained by Madatyan in [11]. He proved that, for each read-once network S over the basis $B(2)$, the minimum cardinality of a diagnostic test is at most $\frac{3}{2}L(S)$. It is easy to see that the minimum depth of a decision tree solving the problem of diagnosis is at most the minimum cardinality of a diagnostic test. From here it follows that the upper bound $h(S) \leq \frac{3}{2}L(S)$ is true for any read-once network S over the basis $B(2)$. Madatyan also showed that, for $k = 1, 2, \ldots$, there exists a read-once network over the basis $\{S_1^5\}$ (see Table 1 for the corresponding figure) with $4k + 1$ edges for which the minimum cardinality of a diagnostic test is equal to $3 \cdot 2^{k+1} - 4$. Note that from the results obtained in this paper it follows that the minimum depth of a decision tree for diagnosis of the considered network is at most $7k$. We see here huge adaptivity gap between complexity of diagnostic decision trees which can be interpreted as adaptive algorithms (the choice of the next query depends on the results of the previous queries) and diagnostic tests which can be interpreted as non-adaptive algorithms (the sequence of queries is fixed).

The second direction is the study of diagnostic tests and decision trees for constant faults in iteration-free combinatorial circuits (read-once formulas) over finite bases P of Boolean functions. From the results obtained by Moshkov in [13] (see also [14]), which generalize essentially the results presented by Goldman and Chipulis [7], and by Karavai [8], it follows that if P contains only linear or only unate Boolean functions then the minimum depth of decision trees in the worst case has at most linear growth depending on the number of gates in the circuits, and it grows exponentially if P contains a non-linear function and a function which is not unate.

The third direction is connected with two problems of learning:

(i) For a given read-once contact network with n edges, using only membership queries we should find a read-once network with n edges implementing the same function as the initial network;
(ii) For a given read-once formula with n variables over a basis P of Boolean functions using only membership queries, we should find a read-once formula over P with n variables that is logically equivalent to the initial formula.

The considered problems are more complex than the problems of diagnosis of constant faults for read-once contact networks or for read-once formulas. The algorithm for solving the problem (i) proposed by Raghavan and Schach in [17] and the algorithm for solving the problem (ii) with $P = \{x \vee y, x \wedge y\}$ proposed by Angluin, Hellerstein, and Karpinski in [3], both use $O(n^2)$ membership queries, in contrast to the linear algorithm presented here.

This paper consists of five sections. In Section 2, we consider main notions and prove a simple lower bound on the depth of decision trees for fault diagnosis. In Section 3, we study the problem of diagnosis of read-once networks over an arbitrary basis B. In Section 4, we consider upper and lower bounds for the parameters $h(Q)$ and $t(Q)$ for each nontrivial indecomposable network Q with at most 10 edges. Section 5 contains short conclusions.

2. Main notions

In this section, we consider main notions connected with networks, constant faults and decision trees for fault diagnosis. We also prove a simple lower bound on the depth of decision trees for fault diagnosis.

A *network* is an undirected graph S with multiple edges and without loops in which two different nodes called *poles* are fixed. For each edge e in S, there is a *simple* path (without repeating nodes) between poles which contains e. We will assume that the set of edges of S is ordered. We denote by $L(S)$ the number of edges in the network S. A network containing only two nodes and one edge connecting these nodes is called *trivial*.