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On the Canonical Representation of Order 3 Discrete Phase Type Distributions[4]

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Abstract

In spite of the fact that discrete phase type (DPH) distributions are used almost as often as continuous phase type (CPH) distributions canonical representation is not available for general (cyclic) order 3 DPH distributions yet.
In this paper we investigate the canonical representation of DPH distributions of order 3. During the course of this investigation we find that the problem of canonical representation of order 3 DPH distributions is far more complex than the one of order 3 CPH distribution. As a result we needed to distinguish 8 different subclasses of order 3 DPH distributions, while it was enough to distinguish 3 subclasses of order 3 CPH distributions for their canonical representation. Additionally, we were not able to prove all subclasses of DPH distributions with the relatively simple methodology which was sufficient for the canonical representation of order 3 DPH distributions.

Keywords: Discrete phase type distributions, Canonical representation, Similarity transformation.

1 Introduction

Stochastic performance models were restricted to "memoryless" distributions (exponential in case of continuous time models and geometrical in case of discrete time

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models) for a long time in order to utilize the nice computational properties of discrete state Markov models. Phase-Type distributions [8,9] have been introduced for relaxing this modeling limitation on the considered distributions, while maintaining the nice Markovian behavior.

For a period of time continuous time stochastic models with CPH distributions were more often applied in performance modeling of computer and communication systems, but also in this period the analysis of the continuous time models were often based on the method of embedded Markov chains, which transforms the analysis problem into discrete time. Later on, with the rise of slotted time telecommunication protocols (e.g., ATM) discrete time models become primary modeling tools (for recent surveys see [1,6]). As a consequence, approximation of experimental data set with CPH gained more attention for a period of time. Especially, the acyclic subset of CPH distributions gained popularity due to the simple canonical forms available for their representation [4]. The use of acyclic PH distributions has a further important consequence. A lot of properties of the acyclic CPH and the acyclic DPH distributions are identical. For example the same canonical representations apply for acyclic DPH distributions as for acyclic CPH ones [3]. Due to this similarity the problem of fitting DPH distributions was considered to be similar to the one of fitting CPH distributions, but this similarity is limited to the acyclic PH distributions only, as it is indicated through a counterexample in [11]. The canonical representation of order 3 CPH distributions is provided in [5]. In this paper we investigate similar canonical forms for order 2 and 3 DPH distributions, which is a much more involved problem. The complexities of the canonical representation of order 3 CPH and DPH distributions are well represented by the number of forms needed to cover the whole order 3 CPH and DPH classes. [5] reports 3 forms which cover the class of order 3 CPH distributions, while here we present 8 forms to cover the class of order 3 DPH distributions.

In a preceding version of this paper [10] we have found canonical forms for DPH distributions of order 3 with all possible eigenvalue structures except one (referred to as PNP case) and presented a conjecture for that case. In the mean time it turned out that the conjecture for the PNP case in [10] was not valid. In this paper we repeat the proved findings of [10] for order 3 DPH distributions and devote special attention to the PNP case. The findings of [10] for order 2 DPH distributions are not presented here.

The rest of the paper is organized as follows. The next section provides a short introduction of DPH distributions. Section 3 summarizes the results of [10] on the canonical representation of DPH distributions of order 3 with all possible eigenvalue structures expect the PNP case. The new results of the paper are presented in Section 4, which discusses the canonical representation of order 3 DPH distributions with PNP eigenvalue structure. The difficulty of the PNP case comes from the fact that the methodology which allowed to prove the canonical forms for order 3 CPH and order 3 DPH with non PNP eigenvalue structure is not applicable for the PNP case.