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Electronic Notes in Theoretical Computer Science 319 (2015) 51–66

Electronic Notes in Theoretical Computer Science

www.elsevier.com/locate/entcs

Open Maps in Concrete Categories and Branching Bisimulation for Prefix Orders

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Abstract

Open maps, as introduced in concurrency theory by Joyal, Nielsen and Winskel, provide an abstract way to define functional bisimulations across a wide variety of models of computation (like labelled transition systems, event structures, etcetera). Furthermore, the existence of a span of open maps characterises the well-known relational definition of bisimulations found in the literature associated with these models of computation. However, in our working category of prefix orders (in which the objects represent the sets of executions generated by arbitrary dynamical systems) the open maps do not immediately result in functional bisimulations and the existence of a span of open maps does not result in an equivalence. This is rather surprising, since prefix orders are mere generalizations of (discrete) execution trees, for which the open map approach is known to work. After taking a closer look at the definition of open map, we show in this paper that the issue can be remedied by considering prefix orders as a concrete category and reinterpreting the definition of open-map in this light. As a bonus, the choice of a path-category on which the notion of open-map relies becomes a natural one, namely the subcategory of embeddings. While the existence of spans still does not result in an equivalence, it is shown that the existence of cospans does. In fact, we present a characterisation of the notion of branching bisimulation by van Glabbeek and Weijland which, to the best of our knowledge, was not studied in the framework of open maps before.

Keywords: Open maps, Prefix orders, Branching bisimulation, Concrete categories.

1 Introduction

Since van Glabbeek's work [20] on comparative concurrency semantics, we are aware of the many ways in which different states of a labelled transition system can be

[1] Thanks goes to Paul Taylor for providing his macro to neatly draw commutative diagrams in LaTeX.

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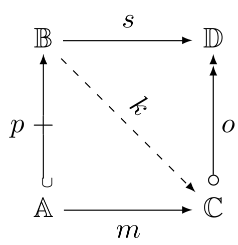
http://dx.doi.org/10.1016/j.entcs.2015.12.005
1571-0661/

considered behaviourally equivalent (resulting in the well-known van Glabbeek spectrum). In their seminal paper [14], Joyal, Nielsen and Winskel proposed an abstract definition of strong bisimulation using the language of category theory and thus embarked a way to capture behavioural equivalences in a uniform framework. In particular, bisimilarity through spans (cospans) of open maps is defined as the existence of a span (cospan) of open maps between two objects, where a map o of $\mathcal{M}$ is *open* (denoted $\circ\!\!\rightarrow\!\!\!\!\twoheadrightarrow$) whenever, for any map p in a subcategory $\mathcal{P}$ (denoted $\hookrightarrow\!\!\!\!\!+\!\!\!\rightarrow$) and maps s and m from $\mathcal{M}$ such that the outer square commutes (i.e., $s \cdot p = k \cdot m$),



there exists a map k in $\mathcal{M}$ (existence emphasised by dashed arrow) making the two inner triangles commute (i.e., $k \cdot p = m$ and $o \cdot k = s$). These arrow-notations will be overloaded later in an obvious way, when discussing the concrete categorical variant of openness.

Taking $\mathcal{M}$ as the category of labeled transition systems with transition preserving maps between them, and $\mathcal{P}$ as the category of path-extensions (containing all transition preserving maps between chains of transitions) Joyal et al. showed that bisimilarity through spans of open maps coincides with the familiar notion of strong bisimulation from concurrency theory [15]. Subsequently, bisimilarity through spans or cospans of open maps has been shown to coincide with useful notions of bisimulation in many alternative models of behaviour as well (see, e.g., [3,9,11,12]).

Despite the generality offered by the open map framework [13], it suffers from two limitations. Firstly, there is as yet no uniform treatment of weak equivalences from the van Glabbeek spectrum (see [17]). Most work on weak equivalences deals with the notion of weak bisimulation (e.g. [3,9]) and seems to rely on first saturating (merging) the so-called *invisible* steps of the transition systems under study and then instantiating the strong bisimilarity on the saturated versions. As it is well known from [19], such a saturation method of the invisible steps is not sound with respect to branching bisimulation equivalence; thus, the techniques developed in [3,9] fall short in characterising branching bisimulation equivalence. Secondly, in order for bisimilarity through spans of open maps to result in an equivalence, the category $\mathcal{M}$ must have pullbacks, which can be a difficult condition to obtain (see [7,16]).

Surprisingly, in our own research [4,5] on describing behavioural systems as prefix ordered sets of executions, the definitions of branching bisimulation arose naturally via a different path, but we had trouble to apply the open map framework of [14] even for strong bisimulation. To be precise, there was no suitable choice of the subcategory of paths such that open maps would result in the usual notion of functional bisimulation (cf. [14, Proposition 1]).