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Alternation-Free Weighted Mu-Calculus: Decidability and Completeness

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Abstract

In this paper we introduce WMC, a weighted version of the alternation-free modal mu-calculus for weighted transition systems. WMC subsumes previously studied weighted extensions of CTL and resembles previously proposed time-extended versions of the modal mu-calculus. We develop, in addition, a symbolic semantics for WMC and demonstrate that the notion of satisfiability coincides with that of symbolic satisfiability. This central result allows us to prove two major meta-properties of WMC. The first is decidability of satisfiability for WMC. In contrast to the classical modal mu-calculus, WMC does not possess the finite model-property. Nevertheless, the finite model property holds for the symbolic semantics and decidability readily follows; and this contrasts to resembling logics for timed transitions systems for which satisfiability has been shown undecidable. As a second main contribution, we provide a complete axiomatization, which applies to both semantics. The completeness proof is non-standard, since the logic is non-compact, and it involves the notion of symbolic models.

Keywords: weighted modal Mu-Calculus, non-compact modal logics, weighted transition systems, satisfiability, complete axiomatization.

1 Introduction

For more than two decades, specification and modelling formalisms have been sought that address essential non-functional properties of embedded and cyber-physical systems. In particular, timed automata [4] were used for expressing and analysing timing constraints of systems with respect to timed logics such as TCTL [3], T_μ [17], L_ν [23] and MTL [19]. However, equally important non-functional properties of embedded or cyber-physical systems are related to consumption of resources, in particular that of energy. This lead initially to weighted extensions of timed automata [5, 6] and most recently to energy automata [9]. However, whereas the

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problems of cost-optimal reachability and infinite runs have been shown to be efficiently computable, the general model checking problem with respect to a weighted extension of TCTL turns out to be undecidable [11].

In this paper, we consider the purely weighted setting, in which the quantitative information of systems is modelled as weighted transition systems (WTSs) with transitions being decorated with non-negative reals besides actions. We study the problems of satisfiability and axiomatization of weighted logics in the most general setting. We develop WMC, a weighted version of the alternation-free modal mu-calculus, that subsumes WCTL and resembles the previously studied timed extension of the modal mu-calculus T_μ and L_ν. WMC is a multi-modal logic with fixed-point operators, where modalities either constrain discrete transitions or the amount of resources in a given state. For the latter, WMC uses resource-variables, similar to the clock-variables used in timed logics, see e.g. [10].

Our first main contribution is to show decidability of satisfiability for WMC. In previous work [27], we proved decidability and finite model property for restriction of WMC with only one resource-variable for each resource and only maximal fixed points. This restriction bounds severely the expressiveness of the logic. In [25, 26], we studied two sub-logics of WMC with multiple resource-variables for each resource and only maximal fixed points. These logics are shown decidable by using the filtration construction, but are significantly weaker than WMC in that resource-variables are restricted to be event-recording. In contrast to these fragments and to modal mu-calculus, WMC does not posses finite model property, thus decidability does not follow from classical arguments. As an alternative, we propose here notions of symbolic model and semantics for which the finite model property does hold. Fortunately – as demonstrated in the paper – the notion of satisfiability coincides with that of symbolic satisfiability, from which our decidability result follows. This should be contrasted to the resembling timed logics for which satisfiability is undecidable.

The fact that the two semantics have the same validities is a remarkable property and a powerful tool that allows us to transport meta-results between the two semantics, in particular computability and complexity results for satisfiability checking and completeness results for proof systems.

Our second main contribution is a complete axiomatization of WMC, allowing all valid properties to be derived as theorems. At the best of our knowledge, this is the first complete axiomatization for a fixed point weighted modal logic in the literature. The axiomatization is remarkably simple, combining modal axioms of non-recursive weighted logic with classic axioms of fixed points [20, 28, 30]. The finite model property provides the arguments to demonstrate that the axiomatization is complete for the symbolic semantics and hence, the completeness result can be extrapolated to the WTS-semantics.

Our third main contribution is the completeness proof itself, which is non-standard and novel in many aspects. Since the logic is non-compact, it requires infinitary