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On Paraconsistent Extensions of C_1

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Abstract

We show that logic C_1 cannot be extended to a paraconsistent logic in which the substitution theorem is valid. We show that C_1 can be extended to larger paraconsistent logics by adding some desirable properties as axioms. We use three-valued logics to support our claims.

Keywords: multi-valued logics, substitution theorem, logic C_1, paraconsistent logic.

1 Introduction

Two main approaches are common to define a logic, the Hilbert axiomatic system and the use of multi-valued tables that define the connectives of the logic. In the first approach the validity of a formula is determined by a set of axioms and a family of inference rules, namely, if the formula can be derived from those axioms and the use of the inference rules, then the formula is valid, otherwise it is not valid. In general, there are many ways of choosing the family of axioms to define a logic and Modus Ponens is one of the most common inference rules appearing in the definition of logics. In the second approach, the tables used to define the logic are called truth

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tables, each connective is regarded as a function taking values in a set of numbers (usually integers) that are specified from the beginning and are called truth values. Some of the values are chosen as designated values. Any formula that evaluates to one of the designated values regardless of the truth values taken by the atoms that appear in the formula, is considered valid. In this paper, we combine both approaches.

In logic, as in any other area of mathematics, when choosing a family of axioms to define a logic, it is desirable to have independence of the axioms, that is, any formula chosen as an axiom should be independent from the other axioms. Multi-valued logics can be used for this purpose (see an example of this in [14]). This methodology sometimes can have limitations (see [10]), however it is useful to researchers interested in the study of logics, such as in our case.

One of the properties we are particularly interested in is paraconsistency. Following Béziau [2], a logic is paraconsistent if it has a negation $\neg$, which is paraconsistent in the sense that the formula $a, \neg a \vdash b$ is not valid, and at the same time has enough strong properties to be called a negation. Paraconsistent logics have important applications, specifically [7] mentions three applications in different fields: Mathematics, Artificial Intelligence and Philosophy. In relation to the second one, the authors mention that in certain domains, such as the construction of expert systems, the presence of inconsistencies is almost unavoidable (see for example [9]). An application that has not been fully recognized is the use of paraconsistent logics in non-monotonic reasoning. In this sense [21,20] illustrate such novel applications.

One example where intuition indicates that paraconsistent logics would be useful for describing abstract structures is provided by Birkhoff and Von Neumann's approach to quantum logic [5].

We emphasize the convenience of accepting local inconsistencies by mentioning Minsky's comment [4] [15]: "*But I do not believe that consistency is necessary or even desirable in a developing intelligent system. No one is ever completely consistent. What is important is how one handles paradox or conflict, how one learns from mistakes, how one turns aside from suspected inconsistencies*". We think that paraconsistent logics could help to give an answer to this important issue addressed by Minsky. In fact, in [16] an interesting approach for Knowledge Representation (KR) was proposed. This approach can be supported by any paraconsistent logic stronger than or equal to C_ω, the weakest paraconsistent logic introduced by Da Costa [8].

Therefore we must consider paraconsistent logics as a supplement to classical logic that deviates from it only in some of its principles (mainly the non-contradiction principle) but that might be applied to contradictory or inconsistent systems like those caused by vagueness or empirical theories whose postulates or basic assumptions are contradictory [5].

Thus, the research on paraconsistent logics is far from being over and, in this work we focus our attention on the paraconsistent logic C_1, which has been studied in [12].

4 "*Minsky's Frame paper*" (1975) in its original form had an appendix entitled "*Criticism of the Logistic approach*"