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Electronic Notes in Theoretical Computer Science 323 (2016) 163–179

Electronic Notes in Theoretical Computer Science

www.elsevier.com/locate/entcs

Multi-focused Proofs with Different Polarity Assignments

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Abstract

In this work, we will reason on how a given focused proof, where atoms are assigned with some polarity, can be transformed into another focused proof, where the polarity assignment to atoms is changed. This will allow, in principle, transforming a proof obtained using one proof system into a proof using another proof system. More specifically, using the intuitionistic focused system LJF restricted to Harrop formulas, we define a procedure, introducing cuts, for transforming a focused proof where an atom is assigned with positive polarity into another focused proof where the same atom is assigned negative polarity and vice-versa. Then we show how to eliminate these cuts, obtaining a very interesting result: while the process of eliminating a cut on a positive atom gives rise to a proof with one smaller cut, in the negative case the number of introduced cuts grows exponentially. We end the paper by showing how to use maximal multi-focusing identify proofs in LJF, giving rise to a 1-1 translation between maximal proofs in LJF and proofs in the natural deduction system for intuitionistic logic NJ, restricted to Harrop formulas.

Keywords: Intuitionistic logic, Proof Systems, Focusing, Identity of proofs.

1 Introduction

In focused proof systems, such as Andreoli's original focused proof system [1] for linear logic or Liang and Miller's LJF and LKF focused proof systems for intuitionistic and classical logics [13], connectives are classified as positive or negative,

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[4] All authors are supported by CNPq. Pimentel is also supported by the project CAPES/FAPERN.

http://dx.doi.org/10.1016/j.entcs.2016.06.011

according to their right introduction rules: positive connectives do not have invertible right rules, while negative connectives are those whose right introduction rules are invertible. The polarity of a non atomic formula is then given by the polarity of its outermost connective. The interesting fact is that atomic formulas can be arbitrarily assigned as positive or negative, without affecting the completeness of the focusing discipline.

While this choice for the polarity of atomic formulas does not affect provability, it does affect the shape of the resulting focused proofs. For instance, in [16] it is shown that, depending on the polarity assignments used for the atomic formula, one can, from the same logical theory, encode sequent calculus or natural deduction proofs. Also, in [6] it has been shown that this choice of polarities can explain different proof search strategies, such as backward chaining and forward chaining. More specifically, focusing and the polarity of atoms were used in order to justify proof theoretically the derivation steps used in the inverse method proof search mechanism. The results are over atoms in Horn theories only.

In this paper we consider a more general setting. In fact, using the focused system LJF [13] for intuitionistic logic restricted to hereditary Harrop formulas [14], we define a procedure, introducing cuts, for transforming a focused proof where an atom is assigned with positive polarity into another focused proof where the same atom is assigned negative polarity and vice-versa. We then show how to eliminate these cuts. Hence, we are able to transform a proof using a forward chaining strategy into a proof using backward chaining strategy or even obtain novel translations from sequent calculus to natural deduction and vice versa.

Interestingly, while the process of eliminating a cut on a positive atom gives rise to a proof with one smaller cut, in the negative case the number of introduced cuts grows exponentially. This difference in the cut-elimination algorithm is most definitely related to the different evaluation strategies according to the Curry-Howard isomorphism, where cut-elimination corresponds to computation in a functional programming setting. We plan to investigate this better in the future.

Finally, we propose a new *multi-focused* system for intuitionistic logic, mLJF, and show how to identify proofs in this system modulo permutations, obtaining the so called *maximal multi-focused* proofs [5,4]. It turns out that such maximal multi-focused proofs, when restricted to Harrop formulas, have a very interesting behavior: if atoms are restricted to the negative polarity, mLJF collapses to LJF, while if atoms are restricted to the positive polarity, for each provable sequent in LJF there is exactly one maximal proof. This means that a proof with negative atoms correspond to a proof with positive atoms and the correspondence is 1-1 up to permutation of rules. This way we provide the first correspondence between an intuitionistic focused system with positive atoms and Gentzen's natural deduction system NJ, thus solving completely the problem of identity of proofs in intuitionistic logic in the sequent calculus setting, when restricted to Harrop formulas.

The paper is organised as follows: Section 2 presents the system LJF and the logic programming fragment based on Harrop formulas, LJF_H; Sections 3 and 4 show how to change polarities of atoms in LJF_H (introducing cuts) and how to