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Electronic Notes in Theoretical Computer Science 323 (2016) 181–196

Electronic Notes in Theoretical Computer Science

www.elsevier.com/locate/entcs

On Strong Normalization in Proof-Graphs for Propositional Logic

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Abstract

Traditional proof theory of Propositional Logic deals with proofs whose size can be huge. Proof theoretical studies discovered exponential gaps between normal or cut free proofs and their respective non-normal proofs. The use of proof-graphs, instead of trees or lists, for representing proofs is getting popular among proof-theoreticians. Proof-graphs serve as a way to study complexity of propositional proofs and to provide more efficient theorem provers, concerning size of propositional proofs.

Fpl-graphs were initially developed for minimal implicational logic representing proofs through references rather than copy. Thus, formulas and sub-deductions preserved in the graph structure, can be shared deleting unnecessary sub-deductions resulting in the reduced proof. In this work, we consider full minimal propositional logic and show how to reduce (eliminating maximal formulas) these representations such that strong normalization theorem can be proved by simply counting the number of maximal formulas in the original derivation. In proof-graphs, the main reason for obtaining the strong normalization property using such simple complexity measure is a direct consequence of the fact that each formula occurs only once in the proof-graph and the case of the hidden maximum formula that usually occurs in the tree-form derivation is already represented in the fpl-graph.

Keywords: Proof Theory, Proof Graphs, N-Graphs, Intuitionistic Logic, Sequent Calculus, Multiple-Conclusion Systems.

1 Introduction

Recently the use of graphs instead of trees to represent proofs has been shown to be more efficient[2][1], while also being helpful to better address the lack of symmetry

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http://dx.doi.org/10.1016/j.entcs.2016.06.012

1571-0661/

in classical ND logic [4] and the complexity of the proof normalization process. Previously we have already presented mimp-graphs as a new proof system developed for minimal implicational logic [6], whose deductions are structured as proof-graph. The point is that in mimp-graphs it is easy to determine maximal formulas [4] and upper bounds on the length of reduction sequences leading to normal proofs. Thus a normalization theorem is proved by counting the number of maximal formulas in the original derivation. The strong normalization property is a direct consequence of such normalization, since any reduction decreases the corresponding measure of derivation complexity. In the present paper we wish to explain this procedure more clearly and expand it onto full propositional logic.

Mimp-graphs are directed graphs whose nodes and edges are labelled. Moreover we distinguish two parts, one representing the inferences of a proof, and the other the formulas. For the formula-part of a mimp-graph, we use directed acyclic graphs, that we denominated formula graphs, consist of basis in the mimp-graph construction and contain only formula nodes sharing formula nodes, thus each formula node only need to occur once in the graph, an example is shown in the left-hand side of Figure 1: the propositions P and Q occur once in the graph.

For the inference-part of a mimp-graph we have the rule nodes (R-nodes) that are labelled by the names of the inference rules. The logic connectives and inference names may be indexed, in order to achieve a 1-1 correspondence between formulas (inferences) and their representations (names), an shown in the right-hand side of Figure 1: the R-node $\rightarrow E_1$ has as major premise the formula graph $(P\rightarrow Q)\rightarrow(P\rightarrow Q)$ and as minor premise the formula graph $P\rightarrow Q$.

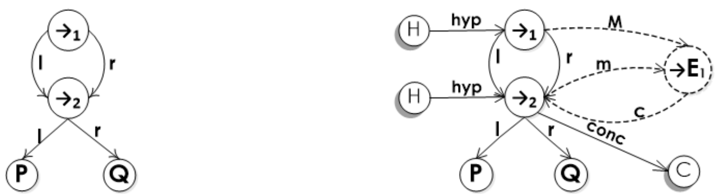

Fig. 1. Formula $(P \rightarrow Q) \rightarrow (P \rightarrow Q)$ depicted as a formula graph (left-hand side) and as major premise of the R-node $\rightarrow E_1$ (right-hand side).

Any strong normalization (SN) proof has to take care of every possible detour (maximal formula) that appears in a derivation. If one considers the normalization as a dynamic process, not all detours that are eliminated in a derivation are explicitly present since the beginning of the process. For example, the permutation-conversions used by Prawitz in the (weak) normalization of intuitionistic logic were designed to take care of hidden maximal formulas (see discussion below). Hiding a detour is a feature of elimination rules similar to ∨. As far as we know, Natural Deduction systems with rules similar to ∨-elimination use permutation-conversions to prove (weak) normalization. SN should deal with these permutation-conversions as well as systems that admit it. Another instance of hidden maximal formula is when after a conversion (reduction), new maximal formulas can appear. This already

[4] A *maximal formula* is a formula occurrence that is consequence of a introduction rule and the major premise of a elimimination rule.