Available online at www.sciencedirect.com

ScienceDirect

Electronic Notes in Theoretical Computer Science 312 (2015) 143–160

Electronic Notes in Theoretical Computer Science

www.elsevier.com/locate/entcs

Practical Extraction of Evidence Terms From Common-knowledge Reasoning

Natalia Novak

Mathematics and Computer Science Department,
Bronx Community College, CUNY,
Bronx NY 10453, USA
Natalia.Novak@bcc.cuny.edu

Abstract

Knowledge, belief, and evidence are fundamental notions which appear in a wide range of areas. Over the last decade epistemic reasoning with justifications has broadened even more the scope of applications of epistemic logic as agents gained the ability to not only reason about epistemic states of knowledge and belief of agents, but also to track their justifications and to sort those which are pertinent to given facts and sufficient for epistemic conclusions.

This paper extends realization algorithm for $\mathsf{S4}$-to-LP case to $\mathsf{S4}_n^{\mathsf{J}}$-to-$\mathsf{S4}_n\mathsf{LP}$ case. It converts cut-free derivations in $\mathsf{S4}_n^{\mathsf{J}}$ into derivations in the corresponding Justification Logic $\mathsf{S4}_n\mathsf{LP}$ where witnesses of knowledge, the justification terms, are recovered for all instances of justified common knowledge. The algorithm was implemented in the MetaPRL framework and was tested on several well-known epistemic puzzles, such as Muddy Children, Surprise Examination Paradox, etc.

Keywords: logic of proofs, logical puzzles, evidence terms, metaprl

Introduction

The study of epistemic reasoning, reasoning about knowledge and belief, is one of the core areas of Computer Science and Artificial Intelligence. The traditional systems of formal epistemology are based on modal logics and have been the subjects of intense research activity during the past decades [10; 15]. There are several computer-aided systems of modal and epistemic reasoning available (for an incomplete list, see [17]).

A foundational effort in this area has enriched modal epistemic logic with the internalized notion of justification, which became part of the language of epistemic logic. This development substantially broadens the scope of applications of epistemic logic. We now have the capability to not only reason about epistemic states of knowledge and belief of agents, but also to track their justifications and to sort those which are pertinent to given facts and sufficient for epistemic conclusions. The very notion of *evidence* has become the subject of rigorous studies.

http://dx.doi.org/10.1016/j.entcs.2015.04.009
1571-0661/

The Artemov's Realization Theorem [2; 3] is the fundamental result that reveals the robust evidence system behind traditional epistemic modal logic reasoning. It recovers evidence terms for each occurrence of epistemic modality in a given theorem. We started with the implementation of improved Artemov's Realization Theorem within the framework of the MetaPRL computer-aided reasoning system, then proceeded with test runs on a wide range of well-known epistemic problems.

1 Translation of $\mathsf{S4}_n^{\mathsf{J}}$ cut-free proofs into $\mathsf{S4}_n\mathsf{LP}$ proof

1.1 *Overview of* $\mathsf{S4}_n\mathsf{LP}$ *logic*

$\mathsf{S4}_n\mathsf{LP}$ [4] is a multi-agent logic of evidence-based knowledge, with knowledge operators of n agents $K_1, K_2, K_3, \dots, K_n$, acting as $\mathsf{S4}$ modalities [10], and evidence assertions of the form $t : A$, where t is an evidence term and A is a formula, as in LP [3]. Evidence term t is built from constants $a, b, c, \dots$ and variables $x, y, z, \dots$ with the help of binary operators '$\cdot$' (application), '$+$' (union), and unary operator '!' (inspection).
Formulas of $\mathsf{S4}_n\mathsf{LP}$ are defined by the following grammar:
$\bot \mid S \mid A \rightarrow B \mid A \wedge B \mid A \vee B \mid \neg A \mid K_i A \mid t : A$, where t is an evidence and S is a sentence variable.

Evidence operation has highest precedence and all other connectives have standard precedence order.

Hilbert-style axioms and rules of $\mathsf{S4}_n\mathsf{LP}$ contain classical propositional logic axioms with the *Modus Ponens* rule along with

Knowledge principles

B1$_i$. $K_i(A \rightarrow B) \rightarrow (K_i A \rightarrow K_i B)$
B2$_i$. $K_i A \rightarrow A$
B3$_i$. $K_i A \rightarrow K_i K_i A$ (*positive introspection*)
R2$_i$. $A \vdash K_i A$ (*knowledge generalization*)
for each individual knowledge operator K_i.

Evidence Principles

E1. $s : (A \rightarrow B) \rightarrow (t : A \rightarrow (s \cdot t) : B)$ (*application*)
E2. $t : A \rightarrow !t : (t : A)$ (*inspection*)
E3. $s : A \rightarrow (s + t) : A, \quad t : A \rightarrow (s + t) : A$ (*union*)
E4. $t : A \rightarrow A$ (*reflexivity*)
R3. $\vdash c : A$, where A is an $\mathsf{S4}_n\mathsf{LP}$ axiom and c is a proof constant (*evidence for axioms*).

Principle connecting evidence and knowledge

C1. $t : A \rightarrow K_i A$ (*undeniability of evidence*).

All axioms are schemas in the language of $\mathsf{S4}_n\mathsf{LP}$. Rules are applied across all sections. The system is closed under substitutions of evidence terms for evidence variables and formulas for propositional variables. Deduction theorem $\Gamma, A \vdash B \Rightarrow \Gamma \vdash A \rightarrow B$ holds, where Γ is a finite set of $\mathsf{S4}_n\mathsf{LP}$ formulas.