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A Note on Finitely Derived Information Systems

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Abstract

The notion of information system initially introduced by Scott provides an efficient approach to represent various kinds of domains. In this note, a new type of information systems named finitely derived information systems is introduced. For this notion, the requirement for the consistency predicate used in Scott's information systems is simplified, and the reflexive and transitive rules for the entailment relation are preserved while the finitely derived rule is introduced. A comprehensive investigation is made on the interrelation between finitely derived information systems and algebraic domains. It turns out that their corresponding categories are equivalent, which indicates that the proposed notion of finitely derived information system provides a concrete approach to representing algebraic domains.

Keywords: Finitely derived information system; Algebraic domain; Equivalence of categories

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1 Introduction

In Domain theory, an interesting topic is to represent various kinds of domain categories by other relatively concrete structures [2]. It can be traced back to the notions of information system and approximable mapping which were initially developed by Scott [7]. Then Larsen and Winskel [5] proved that the category of Scott's informations systems is exactly equivalent to that of Scott Domains (i.e., bounded-complete algebraic domains) with Scott continuous functions being morphisms. Afterwards, Hoofman [3] generalized Scott's information systems to the continuous case and obtained the representation of bounded-complete continuous domains. Recently, Spreen et al. [8] proposed a generalized version of continuous information systems which realizes the representation of continuous domains. In addition, other variations of information systems have been introduced in order to characterize various other kinds of domains [9,10,11].

Theoretically, in Scott's original work, an information system is a triple $(A, Con, \vdash)$ where A is a set, Con is a set of subsets of A and $\vdash$ is a binary relation between Con and A. As advised by Scott, A can be understood as a set of data objects, Con as consistent combinations of data objects and $\vdash$ as an entailment relation which records the dependencies between consistent combinations of data objects and individual ones. As for Con, the following axioms are required: (a) it must be closed under subsets; (b) it contains all singletons; and (c) adjunction of an entailed object to a consistent combination preserves consistency. The entailment $\vdash$ is required to be reflexive and transitive, that is, (i) for any $X \in Con$, if $a \in X$, then $X \vdash a$ (reflexivity); (ii) for any $X, Y \in Con$ and $a \in A$, if $X \vdash Y \vdash a$, then $X \vdash a$ (transitivity).

If we consider A as a set of items (for instance, product items in the supermarket domain), then Scott's information systems may provide a mathematical infrastructure for association rule mining in which Con represents the antecedents of transactions over A and $\vdash$ represents the atomic association rules which indicates the consequents of these antecedents. Nonetheless, we have to be vigilant on the axioms of Scott's information systems in that some of them may be too restrict from the practical viewpoint and, on the other hand, new axioms may be required to reflect some exclusive features of association rules. For instance, the restrictions that Con is closed under subsets and it contains all singletons are obviously unreasonable from the viewpoint of association rule mining. Moreover, we consider that the reflexive and transitive rules for $\vdash$ are necessary to our consideration, but they are not sufficient to appropriately reflect some features of association rule systems. Therefore, we need to adapt Scott's information systems in order to provide a more rational model for association rule mining.

In [4], we proposed the notion of $\mathcal{F}$-context where $\mathcal{F}$ is a finite-subsets-family structure on the attribute set of a formal context. We discussed the implication rule systems induced from consistent $\mathcal{F}$-contexts. The results demonstrate that the concept hierarchy of a consistent $\mathcal{F}$-context is just in correspondence with a special kind of subset family inherent in the induced implication rules system. Based on this result, we introduced the notion of formal implication rule system which