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Formal Contexts for Algebraic Domains

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Abstract

In this paper, we investigate the representation of algebraic domains by means of Formal Concept Analysis. For a formal context, we can define a large number of consistent sets. Associated with each consistent set, there is a set of F-approximable concepts which are selected from the well known approximable concepts. By virtue of F-approximable concepts, formal contexts and algebraic domains are able to interpret each other. Moreover, by analyzing the finitely consistent sets, the algebraic bifinite domains, algebraic $L-$domains are exactly located at the corresponding formal contexts, respectively.

Keywords: formal context, F-approximable concept, algebraic domain, categorical equivalence.

1 Introduction

Domain theory was introduced by Dana Scott [5] in the 1970s and has become an important branch of order theory. As the theory of ordered topological structures used in denotational semantics, it has major applications in computer science, where it is used to specify denotational semantics, especially for functional programming languages.

In order to provide a more concrete way for the usual domain-theoretical approach to the semantics of programming languages, the notion of information system was first developed in [6]. By the notion of information elements, information systems provide a concrete representation of Scott domains. During the last few

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decades, many other kinds of information systems have been developed to represent various kinds of domain structures, such as continuous domains [15], algebraic domains [7], L-domains [8], bifinite domains [2] and so on.

Another research field which is closely related to information systems, and thus related to domain theory, is formal concept analysis (for short, FCA). It was pioneered by Wille [4] and was built on applied lattice and order theory [3]. The core idea of classical FCA is to extract the hierarchy structure of concepts inherent in relational data.

Recently, in [10,13], G-Q. Zhang, etc. studied in detail the connections between information systems and FCA. In their work, based on the observation of the mismatch between the formal concepts and information element of the associated information systems, they initially proposed the notion of formal approximable concept which is a generalization of classical formal concepts. It has been verified that approximable concepts exactly capture the class of algebraic lattices which are special domain structures. Subsequently, P. Hitzler and G-Q Zhang [13] further established the categorical equivalence between a specified category of formal contexts and the category of algebraic lattices. These results indicate the significant potential capability of FCA in characterizing domain structures. However, we have not seen any work on representing domain structures other than algebraic lattices by the tool of FCA in the literature.

In this paper, we investigate the representation of algebraic domains by means of FCA. For this purpose, we propose a new notion of F-approximable concept, which can be viewed as a selection of approximable concepts under the constraint of a consistent set employed on a formal context. We further explore the connection between F-approximable concepts and algebraic domains. It is shown that all F-approximable concepts of a formal context form an algebraic domain, and conversely, every algebraic domain is isomorphic to the set of F-approximable concepts of some special formal context.

The remainder of this paper is organized as follows. In Section 2, we briefly recall some necessary preliminaries of domain theory. In Section 3, we develop the notion of F-approximable concept based on G-Q. Zhang's approximable concepts and obtain the representation theory of algebraic domains via F-approximable concepts.

In Section 4, we propose a new type of morphism, named conditional formal context morphisms, and study the associated category of formal contexts (denoted as **Cct**). We eventually obtain the equivalence of **Cct** and the category of algebraic domains.

Section 5 additionally investigates the corresponding formal contexts of some special algebraic domains. In particular, we present sufficient and necessary conditions of formal contexts to represent algebraic bifinite domains, algebraic L-domains. Section 6 gives conclusions.