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Global and Local Graph Modifiers

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Abstract

We define two modal logics that allow to reason about modifications of graphs. Both have a universal modal operator. The first one only involves global modifications (of some state label, or of some edge label) everywhere in the graph. The second one also allows for modifications that are local to states. The global version generalizes logics of public announcements and public assignments, as well as a logic of preference modification introduced by van Benthem et Liu. By means of reduction axioms we show that it is just as expressive as the underlying logic without global modifiers. We then show that adding local modifiers dramatically increases the power of the logic: the logic of global and local modifiers is undecidable. We finally study its relation with hybrid logic with binder.

Keywords: Public Announcement Logic, graph modifiers.

1 Introduction: modal logic as a language to talk about graphs

Consider a rooted graph, whose nodes are labelled by subsets of a countable set of labels $PROP$, and whose edges are labelled by subsets of a countable set of labels REL. Such a graph can be represented as a 4-tuple $M = \langle W, w, R, V \rangle$ where

- W is a set of states (nodes);
- $w \in W$ is a particular state ('the actual state');

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- $R : REL \longrightarrow 2^{W \times W}$ associates to every edge label a a binary relation on W ('the interpretation of a');
- $V : PROP \longrightarrow 2^{W}$ associates to every state label p a subset of W ('the interpretation of p').

The language of modal logic is a tool to talk about such labelled graphs. The formulas φ and the modalities α are defined by the BNFs

$$\varphi ::= p \mid \neg\varphi \mid \varphi \vee \varphi \mid [\alpha]\varphi$$

$$\alpha ::= a \mid \alpha;\alpha \mid \varphi?$$

where p ranges over $PROP$ and a over REL.

Given a rooted graph (alias a model) $M = \langle W, w, R, V\rangle$, we say that $p \in PROP$ holds in M (noted $M \models p$) if and only if $w \in V(p)$; and a disjunction $\varphi \vee \psi$ holds in M ($M \models \varphi \vee \psi$) iff φ holds in M ($M \models \varphi$) or ψ holds in M ($M \models \psi$). The other boolean operators can be defined likewise. For the modal operators we have the standard definition of truth of a formula in a model:

$$M \models [a]\varphi \text{ iff } \langle W, w', R, V\rangle \models \varphi, \text{ for every } w' \text{ such that } \langle w, w'\rangle \in R(a)$$

Given $M = \langle W, w, R, V\rangle$, we say that the set $\|\varphi\|_M = \{w' \mid \langle W, w', R, V\rangle \models \varphi\}$ is the interpretation of the formula φ.

As has been pointed out by many authors, modal languages are quite poor as compared to that of predicate logic: we cannot directly talk about states in the language, and we can only quantify in a restricted way. The benefit of that restriction is that modal logics are 'so robustly decidable' [14]. The aim of this paper is to study how the basic modal language can be extended in order to talk not only about graphs, but also *modifications* of graphs. To that aim we shall introduce new modalities into the above basic language, whose semantics will be in terms of graph modifications.

Generally speaking, one can think of the following graph modifications: add or delete a state, add or delete a state label of some state, or add or delete an edge label between some states. In the sequel we shall consider all these operations, where the label modifications will come in a global and in a local version. The *modification of state labels* can be done by adding particular action expressions to the language, viz. assignments of the form $p := \varphi$. Such an assignment stipulates that the interpretation of p is modified such that it now matches the last interpretation of φ: the assignment $p := \varphi$ transforms $M = \langle W, w, R, V\rangle$ into $M' = \langle W, w, R, V'\rangle$, where $V'(q) = V(q)$ for $q \neq p$, and $V'(p) = \|\varphi\|_M$. This has been studied in *PDL* and more recently by van Ditmarsch et al. [13]. The *modification of edge labels* can be said to be the topic of the family of dynamic epistemic logics [11]. In the simplest case, public announcements are added to the language. For example in Kooi's logic [5], an announcement $\varphi!_K$ eliminates all those edges leading to states that are not in the interpretation of φ: $\varphi!_K$ transforms $M = \langle W, w, R, V\rangle$ into $M' = \langle W, w, R', V\rangle$, where $R'(a) = R(a) \cap (W \times \|\varphi\|_M)$ for every label $a \in REL$. In public announcement logic *PAL* it is not only the edges leading to $\neg\varphi$-states that are eliminated,