Available online at www.sciencedirect.com
ScienceDirect
Electronic Notes in Theoretical Computer Science 210 (2008) 33–48
Electronic Notes in Theoretical Computer Science
www.elsevier.com/locate/entcs

A Quantum Game Semantics for the Measurement Calculus

Yannick Delbecque[1]

School of Computer Science
McGill University
Montreal, Canada

Abstract

In this paper we present a game semantics for a quantum programming language based on a new definition of quantum strategies. The language studied is MCdata, a typed version of the measurement calculus recently introduced by Danos et. al. We give a soundness and adequacy result based on our quantum game semantics. The main contribution is not the semantics of MCdata but rather the development of ideas suitable for a game theoretic treatment of quantum computation in general.

Keywords: Quantum programming languages, game semantics, quantum games.

1 Introduction

The subject of quantum programming languages has emerged as a new field lying at the intersection of programming languages and quantum computation. The goal is not so much the search for the right notation and semantics, but rather the search for *structure* in quantum computation. Thus the standard programming language ideas of compositionality and modularity emerge in new settings. Quantum computation has some radically new features: the impossibility of copying and of unrestricted discarding, entanglement and superposition and the probabilistic nature of measurement. These features make for entirely new challenges in the search for structure. The field of quantum programming languages is the search for this structure.

The most notable contributions are due to Selinger [14,13] and Abramsky and Coecke [1]. Dealing with higher-order programming languages in the quantum setting has proved to be problematic [15], though a few proposals are emerging [18,16].

[1] Email: yannick.delbecque@mail.mcgill.ca

1571-0661 © 2008 Published by Elsevier B.V. Open access under CC BY-NC-ND license.
doi:10.1016/j.entcs.2008.04.016

Denotational semantics for the various quantum programming languages studied in recent years use the natural setup of density matrices and superoperators [14]. While these are well understood mathematically and physically, they do not provide a satisfactory treatment of higher-order quantum programming languages like those of Selinger and Valiron [16] or van Tonder [18]. The problem is that the obvious category with superoperators as morphisms is not closed: there is no object that correspond to the set of superoperators. Some approaches to find a better category where higher-order programming languages could properly be interpreted have been explored, but to this point, all have failed to produce an adequate category [15].

In this paper, we describe an approach to game semantics for quantum programming languages. Classical game semantics is used to construct tight denotational models of various programming languages and logics. Interest in game semantics for programming languages was sparked in the mid-90s by the introduction of two game-based fully-abstract models for PCF [2,9], after many unsuccessful attempts to construct such tights models for this language using other structures. The approach has since been successfully employed to provide fully abstract models for many other languages with various features (non-determinism, probabilistic, concurrency, etc.), all these results following a similar pattern, each new feature being captured with the help of new types of strategies. This paper introduces a concept of quantum strategy which is conceptually close to these various classical game semantics.

We analyse in detail a game semantics for a particular language: the measurement calculus of Danos et al. [5] which is based on the one-way model of Raussendorf and Briegel [12]. This language is quite low level and quite specific to the one-way model. However, it is a rather novel model of quantum computation and one which has attracted interest among physicists as a basis for implementations. In particular, measurements play a fundamental role, as the name would suggest, and game semantics for this model could shed light on the connection with interpretations of quantum mechanics, for example the consistent histories interpretation [7,11,6]. Our ultimate goal is the development of a higher-order quantum programming language informed by the theory of game semantics. Our work takes *probabilistic game semantics*, as introduced in Danos and Harmer [4], as the starting point, but defines games in terms of quantum ingredients like projective measurement operators.

2 Quantum strategies

The main problem in the interpretation of a quantum language using quantum games and strategies is to find an appropriate quantum version of the classical game semantics definition. The definition of quantum games given below is quite different from what one can find in the literature on quantum games, for example in [10]. In that body of papers, the aim is to generalise probabilistic von Neumann games by letting the players use quantum strategies; this usually creates new Nash equilibriums with better payoffs for the players. These quantum strategies are described as *generalisations* of classical probabilistic strategies. The definition of

Download English Version:

https://daneshyari.com/en/article/423300

Download Persian Version:

https://daneshyari.com/article/423300

Daneshyari.com