Available online at www.sciencedirect.com

ScienceDirect

Electronic Notes in Theoretical Computer Science 322 (2016) 181–195

Electronic Notes in Theoretical Computer Science

www.elsevier.com/locate/entcs

Bounds and Fixed-Parameter Algorithms for Weighted Improper Coloring

Bjarki Agust Gudmundsson [1] Tomas Ken Magnusson [2]
Bjorn Orri Saemundsson [3]

ICE-TCS, School of Computer Science
Reykjavík University
Reykjavík, Iceland

Abstract

We study the weighted improper coloring problem, a generalization of defective coloring. We present some hardness results and in particular we show that weighted improper coloring is not fixed-parameter tractable when parameterized by pathwidth. We generalize bounds for defective coloring to weighted improper coloring and give a bound for weighted improper coloring in terms of the sum of edge weights. Finally we give fixed-parameter algorithms for weighted improper coloring both when parameterized by treewidth and maximum degree and when parameterized by treewidth and precision of edge weights. In particular, we obtain a linear-time algorithm for weighted improper coloring of interval graphs of bounded degree.

Keywords: graph coloring, improper coloring, defective coloring, weighted improper coloring, coloring bounds, fixed-parameter algorithms

1 Introduction

Graph coloring is a classic subject of both mathematics and computer science with many practical applications. It was one of Karp's 21 original $\mathcal{NP}$-complete problems. Many generalizations of graph coloring, such as defective coloring, have been studied, but most of those apply to undirected graphs. In this paper we consider weighted improper coloring, a generalization of graph coloring for weighted digraphs. Weighted improper coloring has not received much attention until recently in the context of wireless networks. In some models for wireless networks, such as the SINR model, communication over one wireless link can disturb communication over other wireless links. This disturbance can vary from link to link, and depends on

[1] Email: bjarkig12@ru.is

[2] Email: tomasm12@ru.is

[3] Email: bjorn12@ru.is

http://dx.doi.org/10.1016/j.entcs.2016.03.013
1571-0661/

signal strength and other variables of the surrounding environment. Furthermore the disturbance does not need to be symmetric. A scheduling of communications in such a network can be modeled as a weighted improper coloring, and this is where the generalization stems from.

In this paper we provide some new bounds, fixed-parameter algorithms and hardness results for weighted improper coloring, some of which are generalizations of existing results for defective coloring.

1.1 Preliminaries

Let $G = (V, E)$ be an undirected graph. For a vertex $v \in V$ let $d(v)$ denote the degree of v in G, and $d_S(v)$ denote the degree of v in the subgraph induced by a subset $S \subseteq V$. A *k-coloring* $c : V \to \{1, \ldots, k\}$ of G is a partition of the vertex set V into k vertex-disjoint subsets, and $c[v]$ denotes the set of vertices that have color $c(v)$. A k-coloring c is *d-defective* if for each $v \in V$, $d_{c[v]}(v) \leq d$. A d-defective k-coloring is also called a *(k, d)-defective coloring.* Note that ordinary coloring is a special case of defective coloring where $d = 0$, so defective coloring is a proper generalization of ordinary coloring.

Let $G = (V, E, w)$ be a weighted digraph, where $w : E \to [0, 1]$ is an associated weight function. For a vertex $v \in V$ let $d^-(v) = \sum_{(u,v)\in E} w(u, v)$ denote its weighted indegree, and $d^-_S(v)$ denote the weighted indegree of v in the subgraph induced by a subset $S \subseteq V$. Let Δ^- denote maximum of any weighted indegree.

Definition 1.1 A k-coloring c of a weighted digraph $G = (V, E, w)$ is a *weighted improper k-coloring* if for each vertex $v \in V$, $d^-_{c[v]}(v) < 1$. The weighted improper chromatic number $\chi_w(G)$ is the minimum number k such that G has a weighted improper k-coloring with respect to the weight function w.

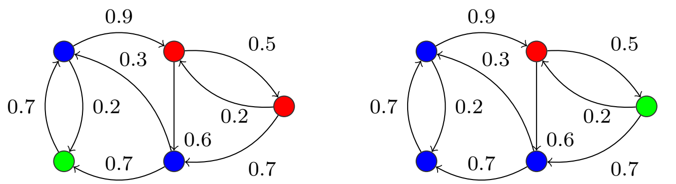

Fig. 1. A valid weighted improper coloring of the graph to the left. The graph on the right has an invalid coloring as the upper left corner vertex has indegree 1 from same-colored neighbors.

Now notice that weighted improper coloring is a generalization of defective coloring, and by extension a generalization of ordinary coloring, as is captured by the following lemma.

Lemma 1.2 *Defective (k, d)-coloring can be reduced to weighted improper k-coloring in polynomial time.*

Proof. Let G be an undirected graph. In a valid defective (k, d)-coloring of G any vertex $v \in V(G)$ can have at most d adjacent vertices of the same color. Let G' be the weighted digraph obtained by replacing every edge $\{u, v\} \in E(G)$ with two