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# Constraining Cycle Alternations in Model Checking for Interval Temporal Logic

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**Abstract**

Model checking is one of the most successful techniques in system verification. While a variety of methods and tools exist to check properties expressed in point-based temporal logics, like LTL and CTL, model checking for interval temporal logic has entered the research agenda only very recently. In previous work, we devised a non-elementary model checking procedure for Halpern and Shoham's modal logic of time intervals, interpreted over finite Kripke structures, and an EXPSPACE algorithm for two meaningful fragments of it. In this paper, we show that the latter algorithm can be suitably tailored in order to check a *subset of the computations of a system*, that satisfy a given bound on the number of cycle alternations, by making use of a *polynomial* (instead of exponential) *working space*. We also prove that such a revised algorithm turns out to be complete for Kripke structures whose strongly connected components are simple cycles.

*Keywords:* Interval Temporal Logic, Model Checking, Computational Complexity

## 1 Introduction

Model checking is one of the most effective techniques in system verification, that allows one to verify a formal specification of the desired properties of a system against a model of its behavior. It has been widely and systematically investigated in the context of classical, point-based temporal logics, whereas it is still almost unexplored in the interval logic setting. In [4,12], the authors propose interval temporal logic (ITL) as a natural and expressive formalism for temporal representation and reasoning. On the one hand, thanks to its high expressiveness (compared to that of standard point-based logic), ITL is well suited for a number of computer science applications, ranging from computational linguistics to formal verification,

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from constraint reasoning to planning [10,11]. On the other hand, undecidability of the satisfiability problem for ITLs is the rule and decidability the exception.

Halpern and Shoham's modal logic of time intervals (HS, for short) is probably the most famous logic among ITLs [4]. It features one modality for each of the 13 possible ordering relations between pairs of intervals (the so-called Allen's relations [1]), apart from the equality relation. The satisfiability problem for HS, interpreted over all relevant (classes of) linear orders, is highly undecidable. Moreover, undecidability rules also over HS fragments; luckily, meaningful exceptions exist, including the interval logic of temporal neighbourhood and the temporal logic of sub-intervals [3].

In this paper, we focus our attention on the model checking problem for HS and its fragments [5,6,7,8,9], for which little work has been done, if compared to LTL or CTL model checking. In the classical formulation of model checking, systems are modelled as (finite) labelled state-transition graphs (Kripke structures), and point-based temporal logics are used to analyse, for each path in the graph, how proposition letters labelling the states change from one state to the next one along the path. In HS model checking, to verify interval properties of computations, we interpret each finite path of a Kripke structure (track) as an interval, whose labeling is defined on the basis of that of the states composing it.

In [5,6], Lomuscio and Michaliszyn address the model checking problem for some HS fragments, extended with epistemic operators. In [5], they focus their attention on the fragment HS$[B, E, D]$ of Allen's relations *started-by*, *finished-by*, and *contains* extended with epistemic modalities. They consider a restricted form of model checking, that verifies a specification against a single (finite) initial computation, and prove that it is a PSPACE-complete problem. In addition, they show that the problem for the purely temporal fragment of the logic is in PTIME. In [6], they prove that the model checking problem for the fragment HS$[A, \overline{B}, L]$ of Allen's relations *meets*, *starts*, and *before*, extended with epistemic modalities, is decidable in non-elementary time. The radically different complexity of the two fragments is not surprising, as the latter allows one to access infinitely many intervals.

In [7,9], Montanari et al. characterize the model checking problem for full HS, interpreted over finite Kripke structures. As in [5,6], formulas of HS are evaluated over finite paths/tracks obtained from the unravelling of a finite Kripke structure. However, in [7,9] a proposition letter holds over an interval (track) if and only if it holds over all its states (homogeneity principle), while in [5,6] truth of proposition letters is defined over pairs of states (the endpoints of tracks/intervals). This makes it difficult to compare the two research contributions. In [9], the authors introduce the basic elements of the picture, namely, the interpretation of HS formulas over (abstract) interval models, the mapping of finite Kripke structures into (abstract) interval models, the notion of track descriptor, and a small model theorem proving (with a non-elementary procedure) the decidability of the model checking problem for full HS against finite Kripke structures. In [7], Molinari et al. work out such a proposal in all its technical details, and they prove that the problem is EXPSPACE-hard. In [8], we consider two large HS fragments, namely, HS$[A, \overline{A}, B, \overline{B}, \overline{E}]$ of Allen's