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Conditional Monte Carlo With Intermediate Estimations for Simulation of Markovian Systems

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Abstract

For systems that are suitable to be modelled by continuous Markov chains, dependability analysis is not always straightforward. When such systems are large and complex, it is usually impossible to compute their dependability measures exactly. An alternative solution is to estimate them by simulation, typically by Monte Carlo simulation. But for highly reliable systems standard simulation can not reach satisfactory accuracy levels (measured by the variance of the estimator) within reasonable computing times. Conditional Monte Carlo with Intermediate Estimations (CMIE) is a simulation method proposal aimed at making accurate estimations of dependability measures on highly reliable Markovian systems. The basis of CMIE is introduced, the unbiasedness of the corresponding estimator is proven, and its variance is shown to be lower than the variance of the standard estimator. A variant of the basic scheme, that applies to large and highly reliable multicomponent systems, is introduced. Some experimental results are shown.

Keywords: dependability, simulation, rare event, Conditional Monte Carlo.

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1 Introduction

We consider systems which can be modelled by a continuous time homogeneous Markov chain X irreducible on the finite state space S (see [5,10,11] or Chapter 6 in [22]). The chain can also be absorbing and the techniques described here still work, but they are easier to present in the irreducible case. In this context, some measures of dependability need the evaluation of the probability $\gamma = \mathbb{P}\{\tau_D < \tau_{\mathbf{u}}\}$, where the times $\tau_{\mathbf{u}}$ and τ_D are defined as follows. The state space of the Markov chain is partitioned as $S = U \cup D$, such that in U the system is *up* and in D the system is *down*. The process X starts at some initial state $\mathbf{u} \in U$. Define $\tau_{\mathbf{u}}$ as the *return time to* $\mathbf{u}$, that is, $\tau_{\mathbf{u}} = \inf\{t > 0\colon X(t) = \mathbf{u} \text{ and } X(t^-) \neq \mathbf{u}\}$, and τ_D as the *hitting time of* D, that is, $\tau_D = \inf\{t > 0\colon X(t) \in D\}$.

The simplest and most basic dependability metric is the Mean Time To Failure (MTTF), defined as the expected life–time of the system, that is, the mean time until the system enters the subset D: MTTF $= \mathbb{E}\{\tau_D\}$. This metric admits the well–known representation MTTF $= \mathbb{E}\{\min(\tau_D, \tau_{\mathbf{u}})\}/\gamma$.

Since we focus on the estimation of γ, we can just collapse all D into a single state $\mathbf{d}$, made absorbing. As before, event $\{\tau_{\mathbf{d}} < \tau_{\mathbf{u}}\}$ means that X gets absorbed at $\mathbf{d}$ before coming back to $\mathbf{u}$. For systems with a large (or infinite) number of states, the exact computation of γ is not feasible, and the standard *Monte Carlo* simulation will work, unless $\gamma \ll 1$, in which case we are facing a *rare event* problem, a context in which acceptable values of the estimator's variance can only be achieved at the expense of a very high number of replications. *Monte Carlo* methods must therefore be improved and adapted to address efficiently this *rare event* problem. Research has resulted in a large number of solutions in this regard, most of which derive from two well known families of techniques named, respectively, *Splitting* [7,8,13,19,26] and *Importance Sampling* [4,5,9].

Some applications of *Splitting* in the context of highly reliable systems can be found in [24] and [25], where the *reliability* and availability estimations of repairable systems are analysed using a variant called RESTART. Recently, some results in the context of static systems have been published in [3] and [12]. Some methods derived from *Importance Sampling*, like Zero-Variance [15,16,17] and Cross-Entropy [20,23] have been successfully applied in the simulation of systems affected by *rare events*.

Conditional Monte Carlo [12,21] is a classic variance reduction technique that has not given rise to many methods in the field of rare events applied to reliability estimation. However, some applications can be found in [1,2,6,27], but most of them are aimed at the *rare events* probability estimation in models that deal with heavy–tailed distributions. This article addresses the problem of reliability estimation in the model so far defined and introduces a *Conditional Monte Carlo* simulation scheme, suitable for the estimation of γ.

The rest of this paper is organized as follows. Section 2 shows a basic application of Conditional Monte Carlo on Markovian systems. Section 3, the core of this paper, introduces modifications to the basic Conditional Monte Carlo algorithm, in order