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Matching of Bigraphs

Lars Birkedal[1] Troels Christoffer Damgaard[1]
Arne John Glenstrup[1]

IT University of Copenhagen, Denmark

Robin Milner[2]

University of Cambridge, UK

Abstract

We analyze the matching problem for bigraphs. In particular, we present a sound and complete inductive characterization of matching of binding bigraphs. Our results pave the way for a provably correct matching algorithm, as needed for an implementation of bigraphical reactive systems.

Keywords: Bigraphs, bigraphical reactive systems, matching, complete inductive characterization.

1 Introduction

Over the last decade, a theory of bigraphical reactive systems has been developed [9,13,14]. Bigraphical reactive systems (BRSs) provide a graphical model of computation in which both locality and connectivity are prominent. In essence, a *bigraph* consists of a *place graph*; a forest, whose nodes represent a variety of computational objects, and a *link graph*, which is a hyper graph connecting ports of the nodes. Bigraphs can be reconfigured by means of *reaction rules*. Loosely speaking, a *bigraphical reactive system* consists of set of bigraphs and a set of reaction rules, which can be used to reconfigure the set of bigraphs. BRSs have been developed with principally two aims in mind: (1) to be able to model directly important aspects of ubiquitous systems by focusing on mobile connectivity (the link graph) and mobile locality (the place graph), and (2) to provide a unification of existing theories by developing a general theory, in which many existing calculi for concurrency

[1] Email: `{birkedal,tcd,panic}@itu.dk`
[2] Email: `Robin.Milner@cl.cam.ac.uk`

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and mobility may be represented, with a uniform behavioural theory. The latter is achieved by representing the dynamics of bigraphs by an abstract definition of reaction rules from which a labelled transition system may be derived in such a way that an associated bisimulation relation is a congruence relation. The unification has recovered existing behavioural theories for the π-calculus [9], the ambient calculus [8], and has contributed to that for Petri nets [11]. Thus the evaluation of the second aim has so far been encouraging. In [2], Birkedal et al. initiate an evaluation of the first aim, in particular it is shown how to give bigraphical models of context-aware systems.

As suggested and argued in [9,1,2] it would be very useful to have an implementation of the dynamics of bigraphical reactive systems to allow experimentation and simulation. In the Bigraphical Programming Languages research project at the IT University, we are working towards such an implementation. The core problem of implementing the dynamics of bigraphical reactive systems is the *matching problem*, that is, to determine for a given bigraph and reaction rule whether and how the reaction rule can be applied to rewrite the bigraph. The topic of the present paper is to analyze the matching problem.

In Figure 1 we show several bigraphs. Consider the bigraph named a. It is intended to model two buildings, one belonging to a corporation and one belonging to a consultancy group. Inside the buildings are laptops with data nested inside folders. The nesting structure depicts the place graph. Links are used to name the buildings and, moreover, to model which folders can be shared between the corporation and the consultancy group and inside the corporation. Thus the laptop shown in the middle is intended to belong to a consultant working for the corporation — the consultant has folders with data belonging to the consultancy group (the link shown to the left) and folders with data belonging to the corporation (the link shown to the right). The fact that folders belonging to the corporation should not leave the corporation is expressed by linking those folders to a so-called binding port on the corporation building, indicated by the circle.

The abstract semantic definition of matching, as defined in the theory of bigraphs [9], is roughly as follows (omitting many details): Given a reaction rule with redex R and reactum R' (with R and R' both bigraphs), and a bigraph a (the agent to be rewritten), if $a = C \circ (R \otimes \mathsf{id}_Z) \circ d$, then it can be rewritten to $C \circ (R' \otimes \mathsf{id}_Z) \circ d$. Here $\circ$ denotes composition of bigraphs and Z is the set of global names of d. In other words, if the reaction rule *matches* a, in the sense that a can be decomposed into a context C, redex R and a parameter d, then a can be rewritten.

Consider again the example in Figure 1. There is a reaction rule expressed by the redex R and the reactum R'; the intention of the reaction rule is to allow copying of data between connected folders in the same nesting hierarchy (note the link in R between the two folders and the so-called local name y). The agent a can be written as a composition of C, R and d — formally, $a = C \circ (R \otimes \mathsf{id}_z) \circ d$. Composition works by (1) plugging the roots of R and d into the holes (aka sites) of C respectively R; (2) fusing together the connections between folder and z (in d) and z and folder (in C), removing the name z in the process; (3) fusing together