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Partial Order Reduction for Probabilistic Branching Time

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Abstract

In the past, partial order reduction has been used successfully to combat the state explosion problem in the context of model checking for non-probabilistic systems. For both linear time and branching time specifications, methods have been developed to apply partial order reduction in the context of model checking. Only recently, results were published that give criteria on applying partial order reduction for verifying quantitative linear time properties for probabilistic systems. This paper presents partial order reduction criteria for Markov decision processes and branching time properties, such as formulas of probabilistic computation tree logic. Moreover, we provide a comparison of the results established so far about reduction conditions for Markov decision processes.

Keywords: partial order reduction, Markov decision process, PCTL, model checking, probabilistic visible bisimulation, ample set

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1 Introduction

Model checking is a technique that allows for the fully automatic verification of a property (often specified in a temporal logic) against a system that is modelled as a network of finite-state automata. It allows for the analysis of qualitative properties such as "every request is eventually answered". Following this example, there are systems whose nature may lead to some occasional unanswered request. Consider, for instance, a protocol that attempts to access a lossy medium a bounded number of times after which it aborts. A property like "access is eventually granted" is obviously false. Instead, to ensure quality of service, one would like that access is granted "often enough". For this purpose, model checking has been extended to deal with *quantitative* properties such as "access is eventually granted with at least 99% probability" [12,3]. In this case, systems are modelled as networks of Markov decision processes (MDP for short) [20].

To reason about non-probabilistic systems, a diversity of methods have been devised to tackle the state-explosion problem that arises when the network of automata is composed. A particular approach is partial order reduction [23,17,10,19, etc.] which is based on the observation that the execution order of concurrent operations does not usually change the validity of a property. Therefore, fixing one particular order of interleaving operations (without generating the others) helps to reduce the number of states and transitions that need to be explored while preserving the properties of interest.

Recently, Baier, Größer and Ciesinki [1] and D'Argenio and Niebert [5] developed independently from each other partial order reduction criteria for MDPs that preserve linear time properties, formalized as quantitative $\mathrm{LTL}_{\setminus X}$ properties. Both approaches rely on modifications of Peled's ample set methods [17,13,18].

The main contribution of this paper is the presentation of partial order reduction criteria for verifying *branching time* properties formalized by means of formulas of probabilistic computation tree logic [3]. Our criteria applied to ordinary transition systems reduce to the criteria suggested by Gerth et al. [10] for non-probabilistic branching time properties. Further on, we discuss the connections between the reduction criteria of [5,1] and those presented here and process equivalences (trace distribution equivalence, suitable notions of simulation and bisimulation).

Although the partial order reduction criteria for verifying branching time properties are rather strong and often might lead to a minor savings of states, our contribution has some impact under both practical and theoretical aspects. First, even a reduction that cannot shrink the state space of an MDP but only