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Quadtrees as an Abstract Domain

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Abstract

Quadtrees have proved popular in computer graphics and spatial databases as a way of representing regions in two dimensional space. This hierarchical data-structure is flexible enough to support non-convex and even disconnected regions, therefore it is natural to ask whether this data-structure can form the basis of an abstract domain. This paper explores this question and suggests that quadtrees offer a new approach to weakly relational domains whilst their hierarchical structure naturally lends itself to representation with boolean functions.

Keywords: Weakly relational domains, abstract interpretation, quadtrees, boolean formulae.

1 Introduction

Program analyses based on abstract interpretation require an abstract domain. One of the first domains described was that of polyhedra [9] and recent work has investigated subclasses of polyhedra, referred to as weakly relational domains (examples include [6,15,16,17,21]). The motivation for weakly relational domains is the cost of polyhedral domain operations: weakly relational domains restrict the dependencies between variables that can be expressed in order to achieve tractable domain operations whilst retaining sufficient expressivity to be useful.

This paper proposes a new abstract domain based on the well-known data-structure of quadtrees [11]. The domain belongs to the weakly relational domain

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family, but its representation is not given in terms of linear inequalities. The representation means that disjoint, non-linear and non-convex regions can be represented naturally, but this flexibility comes at a cost.

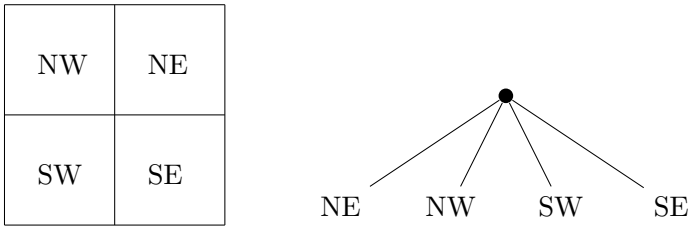
The paper is neutral as to the suitability of quadtrees for use in practical analysers. It is a paper that aspires to promote discussion on the relationship between spatial abstractions and boolean formulae. Nevertheless, the paper makes the following contributions:

- introduces a weakly relational domain for analysis of machine integers that is based on quadtrees
- discusses how this domain might be represented and details how this might be achieved using boolean formulae, either as binary decision diagrams [3] or as formulae in (non-canonical) conjunctive normal form [14]

The paper is structured as follows: section 2 recalls the definition of quadtrees and introduces the underlying idea of using them as an abstract domain; sections 3 and 4 formally introduce the domain and its operations; section 5 discusses the encoding of quadtrees using boolean data-structures and sections 6 and 7 conclude with a survey of related work and a discussion of the strengths and weaknesses of the new domain.

2 Quadtrees

A *quadtree* is a tree where each node has four children; it is interpreted as decomposition of a square in smaller squares, the root being the largest, containing square. A node corresponds to a square and its children to the four squares obtained by dividing the containing square evenly into four. Following [10] the child nodes are ordered anti-clockwise from the top right, as below:



In this work the interest is not only in the decomposition of a square into further squares, but in whether or not these squares are part of some region of interest. Therefore the leaves of quadtrees will be labelled with 0 or 1 to indicate whether or not the corresponding square is part of the region of interest.

Quadtrees are potentially infinite data-structures, as squares can be continually subdivided. However, this work, like others [19], is concerned with analysis over machine integers. This gives a smallest meaningful square, one that is 1×1. Later in this work quadtrees whose smallest square has a larger size will be considered. Henceforth, the minimum square size will be described by the log of its width