Available online at www.sciencedirect.com
ScienceDirect
Electronic Notes in Theoretical Computer Science 72 (2007) 59–74
Electronic Notes in Theoretical Computer Science
www.elsevier.com/locate/entcs

A Duality in Proof Systems for Recursive Type Equality and for Bisimulation Equivalence on Cyclic Term Graphs

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Abstract

This paper is concerned with a proof-theoretic observation about two kinds of proof systems for regular cyclic objects. It is presented for the case of two formal systems that are complete with respect to the notion of “recursive type equality” on a restricted class of recursive types in μ-term notation. Here we show the existence of an immediate duality with a geometrical visualization between proofs in a variant of the coinductive axiom system due to Brandt and Henglein and “consistency-unfoldings” in a variant of a ‘syntactic-matching’ proof system for testing equations between recursive types due to Ariola and Klop. Finally we sketch an analogous result of a duality between a similar pair of proof systems for bisimulation equivalence on equational specifications of cyclic term graphs.

Keywords: Recursive types, recursive type equality, bisimulation, cyclic term graph

1 Introduction

The main part of this paper is concerned with an observation about two complete proof systems for the notion of “recursive type equality” on recursive types.

There are to our knowledge basically two different complete axiom systems known for recursive type equality: (i) A system due to Amadio and Cardelli given in [1] (1993) and (ii) a coinductively motivated axiom system introduced by Brandt and Henglein in [3] (1998). Apart from these axiomatizations, it is also possible to consider (iii) a ‘syntactic-matching’ proof system for which a notion of consistency relative to this system is complete for recursive type equality. Such a system can

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[2] I want to thank J.W. Klop for suggesting this study and for providing me with a couple of initial ideas. And I am also much indebted to Bas Luttik for his careful reading of drafts for this paper. Finally I would like to thank the referees, i.p. for a list with suggestions of specific alterations.

doi:10.1016/j.entcs.2002.09.007

be defined in a very similar way to one that was introduced by Ariola and Klop in [2] (1995) for the notion of bisimulation equivalence on equational representations of cyclic term graphs. For our purpose we will consider only 'normalized' variants without symmetry and transitivity rules of the Brandt–Henglein and syntactic-matching systems. In Section 3 these variant systems will be defined and their respective soundness and completeness theorems stated.

It was noted by Klop that there appears to be a striking similarity between the activities of (a) trying to demonstrate the consistency of an equation between recursive types with respect to the syntactic-matching system and of (b) trying to prove the same equation in the system of Brandt and Henglein. This basic observation underlying the present paper will be described in Section 4.

In order to extract a precise statement from this intuitive recognition, two formal prerequisites turn out to be necessary: Firstly, in Section 5 we will introduce an extension of the variant Brandt–Henglein system with some more coinductive rules. And secondly, in Section 6 we define so called "consistency-unfoldings" of given equations between recursive types in the variant 'syntactic-matching' system as certain formalizations of successful consistency-checks. With these notions our main theorem is stated in Section 7: There exists even a "duality" via easily definable reflection mappings between derivations in the variant Brandt–Henglein system and corresponding consistency-unfoldings in the variant syntactic-matching system.

In Section 8 we furthermore outline an analogous result for a similar pair of proof systems concerned with the bisimulation relation on equational specifications of cyclic term graphs.

2 Preliminaries on recursive types

As in Brandt and Henglein [3] we consider only recursive types denoted by μ-terms in canonical form over the restricted class of finite types with $\to$ as the single type constructor. We assume a countably infinite set *TVar* of *type variables*. The small Greek letters α and β (possibly with subscripts) will be used as syntactical variables for type variables and the letters τ, σ, ρ, χ for recursive types.

Definition 2.1 (Recursive Types in Canonical Form). The set *can-μTp* of *recursive types in canonical form* is generated by the following grammar:

$$\tau \ ::= \ \bot \,|\, \top \,|\, \alpha \,|\, \tau_1 \to \tau_2 \,|\, \mu\alpha.\, \underbrace{(\tau_1 \to \tau_2)}_{\text{where } \alpha \in \text{fv}(\tau_1 \to \tau_2)} \quad . \tag{2.1}$$

By *can-μTp–Eq* we denote the set of all equations $\tau = \sigma$ between recursive types τ and σ in canonical form.

The recursive types in *can-μTp* are in "canonical form" due to the two requirements in the last disjunctive clause in grammar (2.1): For given $\alpha \in \mathit{TVar}$ the μ-operator may only be applied to a previously formed expression τ if τ is of the form $\tau_1 \to \tau_2$ and if α occurs free in $\tau_1 \to \tau_2$. – Our results do not depend on the limitation to consider recursive types *in canonical form* only (cf. forthcoming [4]).