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Phase transitions in two-dimensional daisyworld with small-world effects—A study of local and long-range couplings

Dharani Punithan, R.I. (Bob) McKay *

Structural Complexity Laboratory, Department of Computer Science and Engineering, Institute for Computer Technology, Seoul National University, South Korea

HIGHLIGHTS

- We construct a 2D daisyworld model with small-world effect.
- We use different couplings for temperature and daisies.
- We investigate the role of non-local long-range couplings in daisyworld dynamics.
- We examine the homeostasis emergence of daisyworld.
- We analyse the phase transition region in the small-world regime.

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ABSTRACT

Watson and Lovelock's daisyworld is a coupled biotic–abiotic feedback loop exhibiting interesting planetary ecodynamics. Previous studies have shown fascinating spatio-temporal dynamics in a 2D daisyworld, with the emergence of complex spatial patterns. We introduce small-world effect into such a system. Even a small fraction of long-range couplings destroys the emergent static pattern formation, leading to completely coherent periodic dominance as observed in fully-connected graphs. This change in daisyworld behaviour depends only on the small-world effect, independent of the means by which they are induced (Watts–Strogatz, Newman–Watts and smallest-world models). The transition from static patterns in grid worlds to periodic coexisting dominance in small-worlds is relatively abrupt, exhibiting a critical region of rapid transition. The behaviours in this transition region are a mix of emergent static spatial patterns and large-scale pattern disruption.

1. Introduction

Watson and Lovelock [1] originally proposed the daisyworld model to demonstrate that a biosphere can regulate a planetary environment without external guidance, providing mathematical justification for Lovelock's well-known Gaia hypothesis [2]. This homeostasis emerges from the biosphere's tuning of the planetary albedo (reflectivity). The original daisyworld was modelled on a one-point (0D) world. It was subsequently extended to a one-dimensional (1D) world by Adams et al. [3], to a flat two-dimensional (2D) world by von Bloh et al. [4] and to a curved 2D world by Ackland et al. [5] and Ackland and Wood [6]. Daisyworld's emergent homeostasis has to date been investigated only in simple connection topologies—regular lattices, in which all neighbourhoods are isomorphic, with fixed numbers of nearest neighbours.

However, the graph topologies of the real world are more complex. Watts and Strogatz [7] in their seminal paper proposed that real-world graphs lie between regular and random graphs and are usually highly locally clustered (cliquish), as in regular lattices, yet have low characteristic path lengths, in which they resemble random graphs. This property is known as the "small-world effect". It is found across a wide spectrum of sciences—in social, biological and technological graphs such as neural networks, electronic power grids, networks of movie actors [7], the world wide web [8], scientific collaborations networks [9], and so on.

* Corresponding author.
E-mail address: rimsnucse@gmail.com (R.I. McKay).

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The small-world property was first investigated in the pioneering empirical work of social psychologist Milgram [10]. He showed that any two randomly chosen people on the planet were likely to be linked by a chain of six intermediate acquaintances. His point has been widely accepted, and is referred to as the "small-world phenomenon" (more popularly known as the "six degrees of separation"). More precisely, the underlying social graph is a superposition of high local clustering and short global chains, possessing the small-world property.

Watts and Strogatz [7] (WS) in their ground-breaking work first modelled the small-world effect by randomly rewiring a fraction of the edges of regular lattices. This drastically decreased the diameter of the graph (the longest shortest path between two vertices). Other variants exhibiting the small-world effect include the Newman–Watts (NW) model of Newman and Watts [11] and the Smallest World (SW) model of Dorogovtsev and Mendes [12].

Since a small-world graph realises high local clustering and high connectivity simultaneously, it exhibits properties from regular lattices and random graphs: with regular lattices, it shares local coherence, while with random graphs, it shares rapid diffusion. Although the detailed differences from regular lattices may be small, it can share interesting properties of these two opposed topologies.

In turn, the structural properties of graphs exhibiting small-world effects can have dramatic effect on the dynamics of systems acting on them. Well known examples include the dynamics of epidemics, biological evolution and of diffusion processes. For further reading, see Boccaletti et al. [13].

Motivation. In planetary ecodynamics, some species have a mix of local and global dispersal patterns – for example, coconuts [14] or shellfish [15] – which can dramatically alter their ecological behaviour. In general, in a fixed geosphere, dispersion reduces the likelihood of global extinction of species. However in daisyworld (and according to the Gaia hypothesis, in the real world), the geosphere is not fixed—there is feedback from the biosphere to the geosphere. How do such long-range connections affect the behaviour of a daisyworld? Do they exhibit homeostasis more or less likely?

Contribution of our work. This work is partly an extension of Punithan et al. [16]. We have framed the daisyworld model based on two types of interactions: local couplings using Moore neighbourhood model; and non-local long-range connections (shortcuts) randomly inserted in the underlying lattice according to a specified probability. For the insertion of the long-range connections, we used three different models: WS, NW and SW (see Section 2 for details). We analyse the role of the non-local couplings in the spatio-temporal dynamics and investigate the emergence of homeostasis in a daisyworld with such small-world graph structures.

In the previous daisyworld literature [17, Section 3], the connectivities for both species (black and white), and for the environmental resource (temperature), are the same—local connections. In our model, we use different connectivity topologies. Black and white daisies disperse via both local and long-range connections (by water, air, animal pollinator transport etc.), while temperature diffuses only locally.

Foreshadowing the results, we found that even a few long-range connections dramatically affect the population dynamics of daisyworld, but it is still able to self-regulate the environment (temperature) and thus sustain life (daisies). The introduction of non-local long-range connectivities into a highly locally structured daisy population lattice permits the rapid and homogeneous spread of a dominant daisy type, invading the whole coexisting daisyworld. Although the proportion of long-range connections is small, the pattern of interaction is changed from entirely local [18] to almost entirely global.

If a system exhibits such a dramatic change in its characteristic properties under the perturbations of small changes in some of its model parameters, the system is said to have changed phase. Such phase transitions are very common in physical, social and technical systems. For example, a magnet exhibits magnetism up to a critical temperature, above which the magnetic properties are suddenly destroyed. In the same way, when we perturb the connectivity of our daisyworld graph while leaving all other parameters fixed, it exhibits periodic behaviour while the unaltered system forms static self-organised patterns. One focus of this study is the behaviour in this phase transition.

Outline of the paper. In Section 2, we introduce background material on daisyworld and graph models including small-world graphs. Section 3 describes our specific implementation, combining these ideas. The experimental results of daisyworld simulations on different forms of small-world graphs are presented in Section 4. Discussions based our results and conclusions are represented in Sections 5 and 6 respectively.

2. Background

2.1. Daisyworld model

Daisyworld is a complex interplay of life (black and white daisies) and its environment (temperature). The petal colours (phenotype) of 'temperature-sensitive' daisies influence the planetary albedo (environment-altering trait) via automatic feedback processes. The temperature in turn differentially regulates the daisies' growth rates, changing the planetary albedo. The combined effect of such positive and negative couplings results in global environmental regulation. This emergence leads to an infinite dance of black and white daisies.

The daisyworld system can be constructed on different topologies, for which we now describe some of the background.

2.2. Graph models

A graph is a compact mathematical representation of the connectivity topology of complex systems. It consists of an ordered pair of disjoint sets $G = \langle V(G), E(G)\rangle$ such that $V(G) \neq \phi$ and $E(G) \subseteq \{\{u, v\} : u \neq v \in V(G)\}$, i.e. $E(G)$, the edge set, is a subset of the set of the unordered pairs of a nonempty vertex set $V(G)$ [19], implying that $|E(G)| \leq \binom{n}{2}$; $n = |V(G)|$.[1] G is called *regular* (k-*regular*) if each of its vertices has the same degree (k). It is *fully connected* ($k = n - 1$) if $|E(G)| = \binom{n}{2}$ and random if $kn/2$ out of all possible $\binom{n}{2}$ edges are chosen uniformly randomly with equal probability [20].

2.3. Graph statistics

The most commonly used metrics to characterise structural properties of graphs are the characteristic path length and the clustering coefficient.

Characteristic path length ($\bar{L}$). The characteristic path length is a measure of the global structure of a graph (how well connected a graph is). In general, a path (a walk in which no vertex is visited more than once) from a vertex u to another vertex v in a graph G is a sequence of edges that are traversed from u to v with no edge traversed more than once (trail). There may exist more than one path from u to v in G. The length of a path is the number of edges. The shortest path (or geodesic) between two vertices u and v is the path between them with the smallest length. The

[1] In graph theory, the size of the graph is generally denoted by $|E(G)|$ and the order of the graph by $|V(G)|$; however in complex systems theory, the size usually refers to the vertex set $|V(G)|$.