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Chromatic Number of Resultant of Fuzzy Graphs

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Abstract Fuzzy graph coloring techniques are used to solve many complex real world problems. The chromatic number of complement of fuzzy graph is obtained and compared with the chromatic number of the corresponding fuzzy graph. The chromatic number of the resultant fuzzy graphs is studied, obtained by various operations on fuzzy graphs like union, join and types of products. A solution to routing problem is suggested by using chromaticity of fuzzy graphs.

Keywords Fuzzy graph · Chromatic number · Operations of fuzzy graphs · Routing problem

1. Introduction

Fuzzy graph theory has numerous applications in modern science and technology especially in the fields of information theory, neural networks, expert systems, cluster analysis, medical diagnosis and control theory. Several papers in related areas of fuzzy graph theory are available in literature [1, 2, 5, 8, 10, 15, 16]. Many practical problems such as scheduling, allocation, network problems etc. can be modeled as coloring problems and hence coloring is one of the most studied areas in the research

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of graph theory. Coloring of fuzzy graphs play a vital role in solving complications in networks. A large number of variations in coloring of fuzzy graphs are available in literature. Coloring of fuzzy graphs were introduced by Monoz et al. [12]. The authors have defined the chromatic number of a fuzzy graph $G = (V, \sigma, \mu)$ as a fuzzy subset of V. Eslahchi and Onagh [6] introduced the concept of fuzzy chromatic number $\chi^f(G)$ as the number of partitions of the color classes. Anjaly and Sunitha introduced chromatic number of fuzzy graphs and developed algorithms to the same [4]. Samanta and Pal introduced fuzzy coloring of fuzzy graphs [14].

Even though many papers are available on coloring of fuzzy graphs and its applications, there are no papers known on the relationship between the chromaticity of fuzzy graphs and resultant fuzzy graphs obtained by various fuzzy graph operations. We provide the relationship between chromaticity of fuzzy graph and that of the resultant fuzzy graphs obtained by performing various operations in fuzzy graphs like union, join and different types of products. We also provide an application to illustrate how chromatic number of fuzzy graphs can be used to solve routing problems.

2. Preliminaries

2.1. Some Definitions

Definition 2.1 [11] *A fuzzy graph is an ordered triple $G : (V, \sigma, \mu)$ where V is a set of vertices $\{u_1, u_2, \cdots, u_n\}$, σ is a fuzzy subset of V, i.e., $\sigma : V \to [0, 1]$ and is denoted by $\sigma = \{(u_1, \sigma(u_1)), (u_2, \sigma(u_2)), \cdots, (u_n, \sigma(u_n))\}$ and μ is a fuzzy relation on σ, i.e., $\mu(u, v) \leq \sigma(u) \wedge \sigma(v)\ \forall u, v \in V$.*

We consider fuzzy graph G with no loops and assume that V is finite and nonempty, μ is reflexive (i.e., $\mu(u, u) = \sigma(u), \forall u$) and symmetric (i.e., $\mu(u, v) = \mu(v, u), \forall (u, v)$). In all the examples σ is chosen suitably. Also, we denote the underlying crisp graph of G by $G^* : (\sigma^*, \mu^*)$, where $\sigma^* = \{u \in V \mid \sigma(u) > 0\}$ and $\mu^* = \{(u, v) \in V \times V \mid \mu(u, v) > 0\}$. Throughout we assume that $\sigma^* = V$.

Definition 2.2 [11] *The level set of fuzzy set σ is defined as $\lambda = \{\alpha \mid \sigma(u) = \alpha$ for some $u \in V\}$. For each $\alpha \in \lambda$, G_α denotes the crisp graph $G_\alpha = (\sigma_\alpha, \mu_\alpha)$, where $\sigma_\alpha = \{u \in V \mid \sigma(u) \geq \alpha\}$, $\mu_\alpha = \{(u, v) \in V \times V \mid \mu(u, v) \geq \alpha\}$.*

Definition 2.3 [9] *An arc (u, v) is called M-strong if $\mu(u, v) = \sigma(u) \wedge \sigma(v)$. A fuzzy graph $G : (V, \sigma, \mu)$ is called an M-strong fuzzy graph if $\mu(u, v) = \sigma(u) \wedge \sigma(v) \forall (u, v) \in \mu^*$.*

Definition 2.4 [13] *The complement of a fuzzy graph $G : (V, \sigma, \mu)$ is the fuzzy graph $\overline{G} : (V, \overline{\sigma}, \overline{\mu})$ with $\overline{\sigma}(u) = \sigma(u)$ and $\overline{\mu}(u, v) = \sigma(u) \wedge \sigma(v) - \mu(u, v)$.*

Definition 2.5 *A clique of a simple crisp graph G^* is a subset S of V such that the graph induced by S is complete* [7].

Definition 2.6 [11] *If $G_1 = (V_1, \sigma_1, \mu_1)$ and $G_2 = (V_2, \sigma_2, \mu_2)$ be two fuzzy graphs with $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$, then $G_1 \bigcup G_2$ is the fuzzy graph $(V_1 \bigcup V_2, \sigma, \mu)$, where*

$$\sigma(u) = \begin{cases} \sigma_1(u), & u \in V_1 - V_2, \\ \sigma_2(u), & u \in V_2 - V_1 \end{cases}$$