Fuzzy Inf. Eng. (2015) 7: 317–334

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Fuzzy Information and Engineering
http://www.elsevier.com/locate/fiae

ORIGINAL ARTICLE

Intuitionistic Fuzzy Graphs with Categorical Properties

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Received: 15 May 2014/ Revised: 31 March 2015/
Accepted: 31 July 2015/

Abstract The main purpose of this paper is to show the rationality of some operations, defined or to be defined, on intuitionistic fuzzy graphs. Firstly, three kinds of new product operations (called direct product, lexicographic product, and strong product) are defined in intuitionistic fuzzy graphs, and some important notions on intuitionistic fuzzy graphs are demonstrated by characterizing these notions and their level counterparts graphs such as intuitionistic fuzzy complete graph, cartesian product of intuitionistic fuzzy graphs, composition of intuitionistic fuzzy graphs, union of intuitionistic fuzzy graphs, and join of intuitionistic fuzzy graphs. As a result, a kind of representations of intuitionistic fuzzy graphs and intuitionistic fuzzy complete graphs are given. Next, categorical goodness of intuitionistic fuzzy graphs is illustrated by proving that the category of intuitionistic fuzzy graphs and homomorphisms between them is isomorphic-closed, complete, and co-complete.

Keywords Rationally · Intuitionistic fuzzy graph · Strong intuitionistic fuzzy graph

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Peer review under responsibility of Fuzzy Information and Engineering Branch of the Operations Research Society of China.

http://dx.doi.org/10.1016/j.fiae.2015.09.005

1. Introduction

In 1983, Atanassov [4] introduced the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets. Atanassov added a new component (which determines the degree of non-membership) in the definition of fuzzy sets. The fuzzy sets give the degree of membership of an element in a given set, while intuitionistic fuzzy sets give both the degree of membership and the degree of non-membership which are more-or-less independent from each other, the only requirement is that the sum of these two degrees is not greater than 1. Intuitionistic fuzzy sets have been applied in a wide variety of fields including computer science, engineering, mathematics, medicine, chemistry, economics, etc. Applications of graphs in the mentioned fields are shown in [7-10, 12, 20, 23]. In 1975, Rosenfeld [21] discussed the concept of fuzzy graphs whose basic idea was introduced by Kauffmann [11] in 1973. The fuzzy relation between fuzzy sets were also considered by Rosenfeld and he developed the structure of fuzzy graphs, obtaining analogs of several graph theoretical concepts. And the complement of a fuzzy graph was defined by Mordeson [14].

A fundamental part of intuitionistic fuzzy graphs is, as in the case of classical graph theory, operations on intuitionistic fuzzy graphs. Mordeson and Peng [15] defined some useful operations such as Cartesian product, composition, union and join on fuzzy graphs and gave several deeper and interesting results. Akram and Dudek [2] generalized these operations from fuzzy graphs to interval-valued fuzzy graphs, they also introduced some related notions on interval-valued fuzzy graphs (such as interval-valued fuzzy complete graph and self-complementary interval-valued fuzzy graph). Talebi and Rashmanlou [31] studied properties of isomorphism and complement on interval-valued fuzzy graphs. Likewise, they defined isomorphism and some new operations on vague graphs [32, 33]. Rashmanlou and Jun [18] defined complete interval-valued fuzzy graphs. Samanta and Pal introduced fuzzy planar graphs [24], fuzzy tolerance graph [25], irregular bipolar fuzzy graphs [29], fuzzy k-competition graphs and p-competition fuzzy graphs [27], bipolar fuzzy hypergraphs [26] and investigated several properties. Pal and Rashmanlou [16] studied lots of properties of irregular interval-valued fuzzy graphs. For further details, reader may look into [3, 5, 13, 17-19, 21, 28, 30, 31, 34, 35]. The remaining part of the paper is organized as follows. In Section 2, we introduce some useful preliminary notions and define three kinds of new operations (called direct product, lexicographic, and strong product) on intuitionistic fuzzy graphs. In Section 3, we demonstrate the rationality of some important notions (mainly, operations) on intuitionistic fuzzy graphs, such as intuitionistic fuzzy graphs, intuitionistic fuzzy complete graph, and Cartesian product, direct product, lexicographic product, strong product, composition, union and join of intuitionistic fuzzy graphs, by characterizing these notions by their level counterparts graph. As a result, we give a kind of representation of intuitionistic fuzzy graphs, intuitionistic fuzzy complete graphs. In Section 4, we illustrate categorical goodness of intuitionistic fuzzy graphs by proving that the category of intuitionistic fuzzy graphs and homomorphism between them is isomorphic-closed, complete and co-complete. The final section is concluding remarks.