Fuzzy Inf. Eng. (2015) 7: 369–377

Available online at www.sciencedirect.com

ScienceDirect

Fuzzy Information and Engineering

http://www.elsevier.com/locate/fiae

ORIGINAL ARTICLE

Strong Domination in Fuzzy Graphs

O.T. Manjusha · M.S. Sunitha

Received: 4 November 2014/ Revised: 11 March 2015/
Accepted: 7 April 2015/

Abstract In this paper, the concept of strong domination number is introduced by using membership values of strong arcs in fuzzy graphs. The strong domination number γ_s of complete fuzzy graph and complete bipartite fuzzy graph is determined and bounds is obtained for the strong domination number of fuzzy graphs. Also the relationship between the strong domination number of a fuzzy graph and that of its complement are discussed.

Keywords Fuzzy graph · Strong arcs · Weight of arcs · Strong domination

1. Introduction

Fuzzy graphs were introduced by Rosenfeld [21], who has described the fuzzy analogue of several graph theoretic concepts like paths, cycles, trees and connectedness and established some of their properties [21]. Bhutani and Rosenfeld have introduced the concept of strong arcs [10].

Several works on fuzzy graphs are also done by Akram, Samanta, Dudek, Davvaz, Sunitha, Pal and Pramanik [1-8, 14, 18-20, 22-25, 31, 32]. It was during 1850s, a study of dominating sets in graphs started purely as a problem in the game of chess.

O.T. Manjusha (✉)
Department of Mathematics, Kerala Govt.Polytechnic College, Westhill, Calicut, Kozhikode-673005, Kerala, India
email: manjushaot@gmail.com
M.S. Sunitha
Department of Mathematics, National Institute of Technology, P.O.-NIT, Calicut-673 601, India
Peer review under responsibility of Fuzzy Information and Engineering Branch of the Operations Research Society of China.

http://dx.doi.org/10.1016/j.fiae.2015.09.007

Chess enthusiasts in Europe considered the problem of determining the minimum number of queens that can be placed on a chess board so that all the squares are either attacked by a queen or occupied by a queen. The concept of domination in graphs was introduced by Ore and Berge in 1962 and further studied by Cockayne and Hedetniemi [12]. Domination in fuzzy graphs using effective edges was introduced by Somasundaram and Somasundaram [27]. Domination in fuzzy graphs using strong arcs was discussed by Nagoor Gani and Chandrasekaran [16]. According to them a dominating set of a fuzzy graph $G : (V, \sigma, \mu)$ is a set D of nodes of G such that every node in $V - D$ has at least one strong neighbor in D. Also they defined domination number of G in two ways, one as the minimum number of nodes in any D and the other as the minimum scalar cardinality of any D (i.e., using the weights of nodes in D). In this paper, we define the domination number of fuzzy graph using the weights of strong arcs so as to minimize this parameter further.

This paper is organized as follows. Section 2 contains preliminaries and in Section 3, the strong domination number of a fuzzy graph is defined in a classic way (Definition 3.1). The strong domination number of complete fuzzy graph is the weight of a weakest arc in the fuzzy graph (Proposition 3.1). A necessary and sufficient condition is obtained so that the strong domination number of a fuzzy graph G will be the size of G (Proposition 3.2). An upper bound for the sum of the strong domination number of a fuzzy graph and that of its complement is established (Theorem 3.1). An upper bound for the strong domination number of fuzzy graphs without isolated nodes is obtained (Theorem 3.4). An upper bound for the sum of the strong domination number of a fuzzy graph and that of its complement (with no isolated nodes) is also established (Corollary 3.1). Finally, a lower bound and an upper bound for the strong domination number of fuzzy graphs is obtained by using minimum arc strength, order and maximum strong neighborhood degree (Theorem 3.6).

2. Preliminaries

It is quite known that graphs are simply models of relations. A graph is a convenient way of representing information involving relationship between objects. The objects are represented by vertices and relations, by edges. When there is vagueness in the description of the objects or in its relationships or in both, it is natural that we need to design a 'fuzzy graph model'. We summarize briefly some basic definitions in fuzzy graphs which are presented in [9, 10, 13, 15-17, 21, 26-28].

A fuzzy graph is denoted by $G : (V, \sigma, \mu)$ where V is a node set, σ and μ are mappings defined as $\sigma : V \to [0, 1]$ and $\mu : V \times V \to [0, 1]$, where σ and μ represent the membership values of a vertex and an edge respectively. For any fuzzy graph $\mu(x, y) \leq \min\{\sigma(x), \sigma(y)\}$. We consider fuzzy graph G with no loops and assume that V is finite and non-empty, μ is reflexive (i.e., $\mu(x, x) = \sigma(x)$ for all x) and symmetric (i.e., $\mu(x, y) = \mu(y, x)$ for all (x, y)). In all the examples, σ is chosen suitably. Also, we denote the underlying crisp graph by $G^* : (\sigma^*, \mu^*)$ where $\sigma^* = \{u \in V \mid \sigma(u) > 0\}$ and $\mu^* = \{(u, v) \in V \times V \mid \mu(u, v) > 0\}$. Throughout we assume that $\sigma^* = V$. The fuzzy graph $H : (\tau, \nu)$ is said to be a partial fuzzy subgraph of $G : (\sigma, \mu)$ if $\nu \subseteq \mu$ and $\tau \subseteq \sigma$. In particular, we call $H : (\tau, \nu)$ a fuzzy subgraph of $G : (\sigma, \mu)$ if $\tau(u) = \sigma(u)$ for all $u \in \tau^*$ and $\nu(u, v) = \mu(u, v)$ for all $(u, v) \in \nu^*$. A fuzzy graph $G : (V, \sigma, \mu)$ is