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ORIGINAL ARTICLE

Approximation of Rough Soft Set and Its Application to Lattice

Sankar Kumar Roy · Susanta Bera

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Abstract The approximation of soft set is presented in modified soft rough (MSR) approximation space in this paper, i.e., approximation of an information system with respect to another information one. Besides, the concept of rough soft set is introduced in a modified soft rough approximation space. Various properties are studied like subset, union, intersection on rough soft set with some propositions presented on rough soft set. Moreover, the measure of roughness of soft set is defined in MSR-approximation space and the order relation is introduced on soft set. Furthermore, lattice theory is studied in the MSR-approximation space under a modified rough soft environment. Finally, some realistic examples are considered to usefulness and illustrate of the paper.

Keywords Soft set · Rough set · MSR-approximation space · Lattice

1. Introduction

In recent years, scientists, engineers and mathematicians have shown great interest in uncertainty as it found many fields like decision making, engineering, environmental science, social sciences, and medical science etc. Probability theory, fuzzy set theory

Sankar Kumar Roy (✉) · Susanta Bera (✉)
Department of Applied Mathematics with Oceanology and Computer Programming, Vidyasagar University, Midnapore-721102, West Bengal, India
email: sankroy2006@gmail.com
bera.bapi@gmail.com
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[1], rough set theory [2, 3] and other mathematical tools have been used successfully to describe uncertainty, but each of these theories has its inherent difficulties [4, 5]. Consequently, in 1999, Molodtsov [4] proposed a novel concept for modeling vagueness and uncertainty called soft set theory which is free from difficulties affecting existing methods. The operations of soft set defined by Maji et al. [6] and redefined by Cağman and Enginoglu [7]. Recently, the properties and applications on soft set have been studied increasingly [8-10]. Rough set was initiated by Pawlak [3] as a formal tool for modeling and processing incomplete information in information system. Every rough sets are associated with two crisp sets, called lower and upper approximations and viewed as the sets of elements which certainly and possibly belong to the set. Pawlak's rough set is mainly based on equivalence relation. But, in practical, it is very difficult to find an equivalence relation among the elements of a set. So, some other general relations such as tolerance ones and dominance ones are considered to define rough set models [11, 12]. It has been successfully applied to knowledge discovery, decision analysis, signal processing, mereology and many other fields [13, 14]. Soft set theory and rough set theory are treated as mathematical tools to deal with uncertainty. A connection between these two has been made by Feng et al. [15] and introduced the notion of soft rough set. In this model, they described the parameterize subset on the universe of discourse. As a result, some unusual situations have occurred, like upper approximation of a non-empty set may be empty. Upper approximation of a subset may not contain the set which does not occur in classical rough set theory. To overcome these difficulties, Shabir et al. [14] redefined a soft rough set model called MSR set.

In this paper, we study the approximations of an information system with respect to another information ones. We approximate a soft set with respect a modified soft rough approximation space and introduce the notion of rough soft set. Here, we endeavor to establish link between soft set and rough set in connection with an application in lattice. Also, we introduce the concept of measure of roughness in a soft set and consequently some propositions and examples are presented here.

2. Preliminaries

Definition 2.1 *An information system (or a knowledge representation system) is a pair* (U, A) *of non-empty finite sets* U *and* A *where* U *is a set of objects and* A *is a set of attributes; each attribute* $a \in A$ *is a function* $a : U \to V_a$*, where* V_a *is called set of values of attribute* a*.*

Let U be a non-empty set of universe and R be an equivalence relation on U. The pair (U, R) is called Pawlak's approximation space. The equivalence relation R is often called indiscernibility relation and related to an information system. An indiscernibility relation $R = I(B)$, $B \subseteq A$ is defined as:

$$(x, y) \in I(B) \Leftrightarrow a(x) = a(y), \quad \forall\, a \in B,$$

where $x, y \in U$, and $a(x)$ denotes the value of attribute a for object x.

Using this indiscernibility relation, one can define the following operations as:

$$A_\star(X) = \{x \in U \mid [x]_R \subseteq X\} \text{ and } A^\star(X) = \{x \in U \mid [x]_R \cap X \neq \psi\}.$$