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ORIGINAL ARTICLE

On Nash Equilibrium Strategy of Two-person Zero-sum Games with Trapezoidal Fuzzy Payoffs

Bapi Dutta · S.K. Gupta

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Abstract In this paper, we investigate Nash equilibrium strategy of two-person zero-sum games with fuzzy payoffs. Based on fuzzy max order, Maeda and Cunlin constructed several models in symmetric triangular and asymmetric triangular fuzzy environment, respectively. We extended their models in trapezoidal fuzzy environment and proposed the existence of equilibrium strategies for these models. We also established the relation between Pareto Nash equilibrium strategy and parametric bi-matrix game. In addition, numerical examples are presented to find Pareto Nash equilibrium strategy and weak Pareto Nash equilibrium strategy from bi-matrix game.

Keywords Fuzzy numbers · Two-person zero-sum games · Fuzzy payoffs · Parametric bi-matrix game · Pareto Nash equilibrium strategy

1. Introduction

Game theory is a mathematical tool to describe strategic interactions among multiple decision makers who behave rationale. In 1944, Von Neumann and Morgenstern [1] introduced game theory in their pioneer work "Theory of Games and Economic

Bapi Dutta (✉)
Department of Mathematics, Indian Institute of Technology Patna, Patna-800013, India
email: bapi.iitr@gmail.com
Corresponding Author: S.K. Gupta (✉)
Department of Mathematics, Indian Institute of Technology Roorkee, Roorkee-247667, India
email: skgiitr@gmail.com
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Behavior”. Since then many diverse kinds of mathematical games have been defined with their solution concept proposed. Games are broadly classified into two major categories: cooperative and non-cooperative games. In non-cooperative games, one important class of games are two-person zero-sum matrix games in which we assume that payoffs of outcomes are well defined and certain to both players. But in real-life games, such as economics, finance, business competition, voting, auctions, research and development races cartel behavior, e-commerce etc., the certainty assumption of payoffs is not realistic. Basically in real-life games, players are not able to estimate exactly payoffs of outcomes in the game due to lack of adequate information and/or imprecision of the available information on the environments. For example, different advertising strategies of two competing companies leading to different market shares must be estimated by using approximate values [2]. To overcome this kind of uncertainty and imprecision in available information, Zadeh [3] introduced the concept of fuzzy set theory.

Butnariu [4] introduced the concept of fuzzy sets in non-cooperative game theory for the first time to model each player’s beliefs about action of the other players as fuzzy sets. Later, Billiot [5] extended work of [4]. Buckley [6] investigated a two party non-cooperative games involving both uncertainty and multiple goals. Campos [7] was the first to study non-cooperative matrix games with fuzzy payoffs. Using Yager’s fuzzy numbers ranking index [8], the author in [7] transformed the problem of finding solution to fuzzy matrix game into a pair of fuzzy linear programming problems. In [9, 10], Li proposed a multi-objective programming approach to two-person zero-sum matrix games with triangular fuzzy payoffs. Bector et al. [11] studied matrix game with fuzzy goals. Utilizing fuzzy linear programming duality results, they transformed the finding solution to matrix game into a pair of fuzzy primal-dual linear programming problems. Vijay et al. [12] investigated two-person zero-sum matrix games with fuzzy payoffs and fuzzy goals and by using a suitable defuzzification function, they proved that such a game is equivalent to a primal-dual pair of certain fuzzy linear programming problems in which both goals as well as parameters are fuzzy. Sakawa and Nishizaki [13] studied two-person zero-sum multi-objective matrix games with fuzzy payoffs and fuzzy goals. In [14], Cevikel and Ahlatcioglu presented two models for studying two persons zero-sum games with fuzzy payoffs and fuzzy goals. In [15], Li developed a fast approach for computing fuzzy values of matrix games with payoffs of triangular fuzzy numbers.

The most commonly used solution concept in traditional game theory is that of Nash equilibrium, which has been introduced by Nash [16]. Maeda [17] extended Nash equilibrium solution concept for two-person zero-sum games with fuzzy payoffs. Using α-cut and fuzzy max order, introduced by Ramik [18], Maeda studied three kinds of equilibrium strategies, their existence conditions and relation with parametric bi-matrix games in the setting of symmetric triangular fuzzy number. Han et al. [19] developed Nash equilibrium solution concepts for bi-matrix game with fuzzy payoffs of symmetric triangular fuzzy numbers. Cunlin and Qiang [20] defined the max-order principles for asymmetric fuzzy numbers and extended the Nash equilibrium solution concepts for two person zero sum games with fuzzy payoffs of asymmetric triangular fuzzy numbers. They also introduced two special types of fuzzy