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ORIGINAL ARTICLE

Operator's Fuzzy Norm and Some Properties

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Abstract In this paper, a concept of operator's fuzzy norm is introduced for the first time in general t-norm setting. Ideas of fuzzy continuous operators, fuzzy bounded linear operators are given with some properties of such operators studied in this general setting.

Keywords Fuzzy norm · Fuzzy continuous operator · Fuzzy bounded linear operator

1. Introduction

The problem of defining fuzzy norm on a linear space was first initiated by Katsaras [1] and afterwards C. Felbin [2], Cheng & Mordeson [3], came up with their definitions of fuzzy norms approaching from different perspectives. Some authors worked on related topics in fuzzy setting [4-6]. In [7], we have also taken a definition of fuzzy norm slightly different from that of Cheng & Mordeson with a view to establish a complete decomposition of a fuzzy norm into crisp norms. Interestingly, this decomposition theorem played a crucial role in developing fuzzy functional analysis [8-11]. However, for doing so, we had to restrict the underlying $'t'$-norm in the triangle inequality of fuzzy norm to be the $'t'_{\min}$. This has become a bit of uncomfortable

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situation in the sense that uncertainty processing through fuzzy theory demands as much generality as possible in the underlying t-norm. Because of this we have tried to address this problem in two directions. In [12], this has been taken care of by using the concept of 'generating spaces of quasi-norm family' which plays the role of fuzzy norms in some sense of its decomposition under the general t-norm setting. On the other hand in [13, 14], we have tried to fuzzify the results of finite dimensional normed linear spaces with general t-norm but without using the decomposition technique.

With the latter approach, in this paper, we have been able to proceed further. The concept of fuzzy bounded linear operators, fuzzy continuous operators, fuzzy operator norm for fuzzy bounded linear operators and spaces of fuzzy bounded linear operators are introduced and their properties are studied.

The organization of the paper is as in the following:

Section 2 comprises some preliminary results. Definitions of fuzzy continuous operators, fuzzy bounded linear operators are introduced and relation between them are studied in Section 3. In Section 4, we introduce the idea of operator's fuzzy norm. Lastly in Section 5, completeness of $BF(X, Y)$ (set of all fuzzy bounded linear operators) is proved.

2. Preliminaries

Definition 2.1 [15] *A binary operation* $* : [0,\ 1] \times [0,\ 1] \to [0,\ 1]$ *is a t-norm if it satisfies the following conditions:*

(I) $*$ *is associative and commutative;*

(II) $a * 1 = a \ \ \forall a \in [0,\ 1]$;

(III) $a * b \leq c * d$ *whenever* $a \leq c$ *and* $b \leq d$ *for each* $a,\ b,\ c,\ d \in [0,\ 1]$.

If $*$ *is continuous, then it is called continuous t-norm. Following are examples of some t-norms that are frequently used as fuzzy intersections defined for all* $a,\ b \in [0,\ 1]$.

(I) *Standard intersection:* $a * b = \min(a,\ b)$.

(II) *Algebraic product:* $a * b = ab$.

(III) *Bounded difference:* $a * b = \max(0,\ a + b - 1)$.

(IV) *Drastic intersection:*

$$a * b = \begin{cases} a, & for \quad b = 1, \\ b, & for \quad a = 1, \\ 0, & otherwise. \end{cases}$$

The relations among these t-norms are $a * b$ (Drastic)$\leq \max(0,\ a + b - 1) \leq\ ab \leq \min(a,\ b)$.

Definition 2.2 [13] *Let* U *be a linear space over the field* $\mathcal{F}$ (C *or* $\mathcal{R}$). *A fuzzy subset* N *of* $U \times \mathcal{R}$ ($\mathcal{R}$*- the set of all real numbers) is called a fuzzy norm on* U *if*