Fuzzy Inf. Eng. (2015) 7: 245–255

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Fuzzy Information and Engineering

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ORIGINAL ARTICLE

Connectivity Analysis of Cyclically Balanced Fuzzy Graphs

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Received: 13 November 2014/ Revised: 20 April 2015/
Accepted: 15 May 2015/

Abstract The concepts of connectivity and cycle connectivity play an important role in fuzzy graph theory. In this article, cyclic cutvertices, cyclic bridges and cyclically balanced fuzzy graphs are discussed. It is proved that a vertex in a fuzzy graph is a cyclic cutvertex if and only if it is a common vertex of all strong cycles with maximum strength. Also a cyclic cutvertex cannot be a fuzzy endvertex in a fuzzy graph. A characterization of cyclically balanced fuzzy graphs is obtained. Cyclic vertex connectivity and cyclic edge connectivity of fuzzy graphs are also discussed.

Keywords Fuzzy graph · Cyclic cutvertex · Cyclic bridge · Cyclically balanced fuzzy graph.

1. Introduction

Zadeh introduced the concept of 'fuzzy sets' in 1965 [24]. He gave a new approach to mathematical theory which deals with uncertainty and lack of precision. Basic results in fuzzy set theory and a comparison between classical set theory and fuzzy set theory were given by Zadeh [25].

After Zadeh, the invention of 'fuzzy graphs' by Rosenfeld in 1975 [14] made a revolution in the field of graph theory. An independent study of 'fuzzy graphs' were

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Peer review under responsibility of Fuzzy Information and Engineering Branch of the Operations Research Society of China.

http://dx.doi.org/10.1016/j.fiae.2015.05.008

made by Yeh and Bang in 1975 [23]. Basic definitions on fuzzy sets and fuzzy graphs can be seen in [1-7]. Rosenfeld [5] introduced basic connectivity concepts in fuzzy graphs. Strong arcs and fuzzy endnodes were studied by Bhutani [3, 5]. The complement of a fuzzy graph was introduced by Sunitha and Vijayakumar [22]. Bipolar fuzzy graphs were introduced by Akram [8] and blocks in fuzzy graphs were studied by Mathew and Sunitha [18, 21]. Many related works can be seen in [8-13].

Also in 2009, Mathew and Sunitha [15] classified different types of edges in fuzzy graphs as α, β and δ based on the strengths of connectedness between the end vertices of an edge. They also introduced the concept of fuzzy edge cut and fuzzy vertex cut, which are set of vertices and edges, whose removal from the fuzzy graph reduces the strength of connectedness between some pair of vertices. In 2013, the notion of cycle connectivity was introduced by Mathew and Sunitha [17]. Cycle connectivity is the maximum among the strength of strong cycles passing through different pairs of vertices in a fuzzy graph. Zadeh's fuzzy logic and fuzzy set theory are used widely in fields like artificial intelligence, clustering analysis, database theory, network analysis and pattern recognition [7]. Connectivity also plays an important role in neural networks and clustering problems [16].

2. Preliminaries

A fuzzy graph [14] is a pair $G : (\sigma, \mu)$ where σ is a fuzzy subset of a set S and μ is a fuzzy relation on σ. We assume that S is finite and nonempty, μ is reflexive and symmetric [14]. In all the examples, σ is chosen suitably. Also, we denote the underlying graph by $G^* : (\sigma^*, \mu^*)$ where $\sigma^* = \{u \epsilon S \mid \sigma(u) > 0\}$ and $\mu^* = \{(u, v) \epsilon S \times S \mid \mu(u, v) > 0\}$. A fuzzy graph $H : (\tau, \nu)$ is called a fuzzy subgraph of $G : (\sigma, \mu)$ if $\tau(u) = \sigma(u)$ for every $u \epsilon \tau^*$ and $\nu(u, v) = \mu(u, v)$ for every $(u, v) \epsilon \nu^*$.

A path P of length n is a sequence of distinct vertices $u_0, u_1, \cdots, u_n$ such that $\mu(u_{i-1}, u_i) > 0$, $i = 1, 2, \cdots, n$ and the degree of membership of a weakest edge is defined as its strength. If $u_0 = u_n$ and $n \geq 3$, then P is called a cycle and a cycle P is called a fuzzy cycle, if it contains more than one weakest edge [7]. The strength of connectedness between two vertices x and y is defined as the maximum of the strengths of all paths between x and y and is denoted by $CONN_G(x, y)$. An $x - y$ path P is called a strongest $x - y$ path if its strength equals $CONN_G(x, y)$ [14]. A fuzzy graph $G : (\sigma, \mu)$ is said to be connected if for every x, y in σ^*, $CONN_G(x, y) > 0$. Through out, we assume that G is connected. An edge of a fuzzy graph is called strong if its membership value is at least as great as the connectedness of its end vertices when it is deleted and an $x - y$ path P is called a strong path if P contains only strong edges [5]. The strong degree of a vertex $v \in \sigma^*$ is denoted by $d_s(v)$ is defined as the sum of membership values of all strong edges incident at v. The minimum strong degree of G is $\delta_s(G)$ and the maximum strong degree is $\Delta_s(G)$.

An edge is called a fuzzy bridge of G if its removal reduces the strength of connectedness between some pair of vertices in G. Similarly, a fuzzy cutvertex w is a vertex in G whose removal from G reduces the strength of connectedness between some pair of vertices other than w. A connected fuzzy graph $G : (\sigma, \mu)$ is a fuzzy tree if it has a fuzzy spanning subgraph $F : (\sigma, \nu)$, which is a tree, where for all edges