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Comparison of some descriptional complexities of 0L systems obtained by a unifying approach

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ABSTRACT

We discuss complexity measures which are obtained as norms of vectors whose components are numerical measures of the sets of productions with the same left-hand side. We show that most of the descriptional complexity measures studied hitherto can be covered by this approach. Further we compare some of the measures with each other in the case of 0L systems.

1. Introduction

The study of the efficiency of descriptions of languages by grammars has begun around 1970. It started with the consideration of context-free and regular grammars, and in the sequel almost all types of grammars and systems generating languages have been investigated with respect to descriptional complexity measures. The number of nonterminals or productions or symbols or active symbols and the degree of nondeterminism belong to the measures studied in the past. Let us have a closer look at the number of productions and the degree of nondeterminism. The former measure is the cardinality of the set of productions of the grammar or system, and the latter one is the maximal number of productions with the same left-hand side. This can be reformulated as follows. Let $\underline{p} = (p_1, p_2, \ldots, p_n)$ be the vector, where the components are the cardinalities of the set of productions with the same left-hand side. Then the number of productions is the usual sum norm (or L_1 norm) of the vector $\underline{p}$, and the degree of nondeterminism is the maximum norm (or L_∞ norm) of $\underline{p}$. This observation is the starting point of this paper. We consider a vector of numbers, where the components are numerical measures on the set of productions having the same left-hand side. Then we can associate a complexity with the grammar or the system by taking some norm of the vector (and can extend it to languages in the usual way). We show that many of the measures investigated in the last years can be described in this way, and we get a series of new measures by this method, this way working towards a unifying approach to descriptional complexity.

As a running example, we focus on the descriptional complexity of (interactionless non-tabled) Lindenmayer systems, where we can cover within our framework all measures which can be found in the literature. However, it is straightforward to adapt our terminology to any other type of grammars or of automata. We give a comparison of the different measures obtained as described above, i.e., we discuss two problems for given measures π and π':
– Are there functions f and g such that $\pi'(L) \leqslant f(\pi(L))$ and $\pi(L) \leqslant g(\pi'(L))$ hold for every language L of the class under consideration? Intuitively, this means that we look for functions which bound one measure by the other.

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– Given two numbers m and n (satisfying $n \leqslant f(m)$ and $m \leqslant g(n)$ if functions f and g exist), does there exist a language L such that $\pi(L) = m$ and $\pi'(L) = n$?

The paper is organized as follows. In Section 2, we define six elementary measures for sets of productions and the corresponding complexities obtained by the usual L_i norms. We give their relations to known measures and some elementary facts on these measures. In Section 3, for all our elementary measures μ, we present a comparison of the measures based on the sum norm associated with μ and the maximum norm associated with μ. In Section 4, we compare the measures based on the maximum norm with respect to the different elementary measures.

2. Introducing our framework

For an alphabet V, we denote the set of all (non-empty) words over V by V^* (and V^+, respectively). The length of a word $w \in V^*$ is denoted by $|w|$. For a letter $a \in V$ and a word $w \in V^*$, $\#_a(w)$ denotes the number of occurrences of a in w. Given a language L, we set

$$alph(L) = \{a \mid \#_a(w) > 0 \text{ for some } w \in L\}.$$

An *interactionless L system* (abbreviated as a 0L system) is a triple $G = (V,P,w)$ where V is an alphabet, w is a non-empty word over V and P is a finite subset of $V \times V^*$ such that, for any $a \in V$, there is at least one element (a,v) in P.

The alphabet V and the word w are called the underlying alphabet and the axiom of the system, respectively. The elements (a,v) of P are called productions or rules (for a) and are written as $a \to v$. For a rule $p = a \to v$, we set $lh(p) = a$ and $rh(p) = v$. For $a \in V$, we set

$$P_a = \{p \mid p \in P, lh(p) = a\}.$$

Fix the alphabet order $V = \{a_1,a_2,\ldots,a_r\}$ in what follows. We call the vector $\vec{P} = (P_{a_1},\ldots,P_{a_r})$ the *descriptional basis* of G. We say that $x \in V^+$ directly derives $y \in V^*$, written as $x \Longrightarrow y$, if $x = x_1x_2\ldots x_n$ for some $n \geqslant 1, x_i \in V, 1 \leqslant i \leqslant n, y = y_1y_2\ldots y_n$ and $x_i \to y_i \in P$ for $1 \leqslant i \leqslant n$, i.e., any letter of x is replaced according to the rules of P. Thus the derivation process in a 0L system is purely parallel. By $\Longrightarrow^*$ we denote the reflexive and transitive closure of $\Longrightarrow$. The language $L(G)$ generated by G is defined as

$$L(G) = \{z \mid w \Longrightarrow^* z\}.$$

Moreover, $L_n(G)$ is the set of all words which can be obtained by n iterated applications of $\Longrightarrow$ starting from w. Obviously, $L(G) = \bigcup_{n\geqslant 0} L_n(G)$.

Let $\mathcal{P}$ be a set of rules. An *elementary measure of descriptional complexity* is a mapping $\mu : \mathcal{P} \to \mathbb{N}$; it will be called *standard* if $\mu(\emptyset) = 0$ and $\mu(P_1) \leqslant \mu(P_2)$ for $P_1 \subseteq P_2$.

Let $G = (\{a_1,a_2,\ldots,a_r\},P,w)$ be a 0L system, and let μ be an elementary measure of descriptional complexity. Then we set

$$\mu(G) = \mu(\vec{P}) = (\mu(P_{a_1}),\mu(P_{a_2}),\ldots,\mu(P_{a_r})).$$

Further let $\|\cdot\|$ be a norm defined on the r-dimensional space over $\mathbb{N}$ or $\mathbb{Q}$ or $\mathbb{R}$. Then we define the descriptional complexity $\mu^{\|\cdot\|}$ of G with respect to the elementary measure μ and the norm $\|\cdot\|$ as $\mu^{\|\cdot\|}(G) = \| \mu(G) \|$. Mainly, we are interested in the well-known usual norms

$$\| (m_1,m_2,\ldots,m_r) \|_n = \sqrt[n]{m_1^n + m_2^n + \cdots + m_r^n},$$

where $n \in \mathbb{N}$ or $n = \infty$. Especially, we get the sum norm

$$\| (m_1,m_2,\ldots,m_r) \|_1 = m_1 + m_2 + \cdots + m_r$$

and the maximum norm

$$\| (m_1,m_2,\ldots,m_r) \|_\infty = \max\{m_1,m_2,\ldots,m_r\}.$$

If we use these norms, then we write μ^n instead of $\mu^{\|\cdot\|_n}$.

We extend the complexity measure to languages in the usual way. For a 0L language L we set

$$\mu^{\|\cdot\|}(L) = \min\{\mu^{\|\cdot\|}(G) \mid L(G) = L\}.$$

The following lemma relates two complexity measures of this type.

Lemma 1. *Let $\pi = \mu^{\|\cdot\|}$ and $\pi' = (\mu')^{\|\cdot\|'}$ for some elementary measures μ and μ' and some norms $\|\cdot\|$ and $\|\cdot\|'$. If $\pi(G) \leqslant \pi'(G)$ holds for any 0L system G, then $\pi(L) \leqslant \pi'(L)$ is valid for any 0L language L.*

Proof. Let L be an arbitrary 0L language and G a 0L system such that $L(G) = L$ and $\pi'(L) = \pi'(G)$. Then we have

$$\pi(L) \leqslant \pi(G) \leqslant \pi'(G) = \pi'(L)$$

which proves the statement. □