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A series of slowly synchronizing automata with a zero state over a small alphabet ☆

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ABSTRACT

For each integer $n \geqslant 8$, we construct an n-state synchronizing automation with a zero state and only 2 input letters such that the minimum length of reset words for the automaton is $\left\lceil \frac{n^2+6n-16}{4} \right\rceil$.

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1. Background and motivation

A *deterministic finite automaton* (DFA) $\mathcal{A}$ is a triple $\langle Q,\Sigma,\delta\rangle$, where Q is a finite set of *states*, Σ is a finite *alphabet*, and $\delta : Q \times \Sigma \to Q$ is a totally defined *transition function*. The function δ extends in a unique way to an action $Q \times \Sigma^* \to Q$ of the free monoid Σ^* over Σ; this extension is still denoted by δ.

A DFA $\mathcal{A}$ is called *synchronizing* if there exists a word $w \in \Sigma^*$ whose action resets $\mathcal{A}$, that is, leaves the automaton in one particular state no matter at which state in Q it started: $\delta(q,w) = \delta(q',w)$ for all $q,q' \in Q$. Any word w with this property is said to be a *reset* or *syncronizing word* for the automaton.

Černý [5] constructed for each positive integer n an n-state synchronizing automaton with 2 input letters such that the minimum length of reset words for the automaton is $(n-1)^2$. The famous Černý *conjecture* claims the optimality of this construction, that is, $(n-1)^2$ is conjectured to be the precise value for the maximum length of shortest reset words for synchronizing automata with n states. The conjecture remains open for more than 40 years and is arguably the most longstanding open problem in the combinatorial theory of finite automata.

Upper bounds within the confines of the Černý conjecture have been obtained for the maximum length of shortest reset words for synchronizing automata in some special classes, see, e.g. [7,10,6,8,2,3,4,11]. One of these classes is the class of automata with a zero state. A state z of a DFA $\mathcal{A} = \langle Q,\Sigma,\delta\rangle$ is said to be a *zero state* if $\delta(z,a) = z$ for all $a \in \Sigma$. It is clear that a *synchronizingautomaton* may have at most one zero state and each word that resets a synchronizing automaton possessing a zero state must bring all states to the zero state. We always denote the zero state of a *synchronizingautomaton* by 0 and refer to synchronizing automata with 0 as *synchronizing* 0*-automata*.

A rather straightforward argument shows that every n-state synchronizing 0-automaton can be reset by a word of length $\frac{n(n-1)}{2}$, see, e.g. [10]. This upper bound is in fact tight because, for each n, there exists a synchronizing 0-automaton with n

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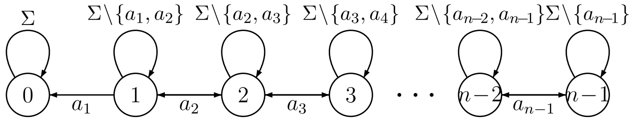

Fig. 1. A 0-automaton whose shortest reset word is of length $\frac{n(n-1)}{2}$.

states and $n-1$ input letters which cannot be reset by any word of length less than $\frac{n(n-1)}{2}$. Such an automaton[1] is shown in Fig. 1.

An essential feature of the example in Fig. 1 is that the input alphabet size grows with the number of states. This contrasts with the aforementioned examples due to Černý [5] in which the alphabet is independent of the state number and leads to the following natural problem: *to determine the maximum length of the shortest reset word for n-state synchronizing 0-automata over a fixed input alphabet*. The most interesting case is a case of 2-letter input alphabet. To the best of our knowledge, all previously known results in the area were consistent with the possibility that this maximum length would behave as a linear function of n. Moreover, such a linear upper bound does exist for so-called monotonic synchronizing 0-automata as follows from a recent result in [1].

The main result of the present paper shows that for general synchronizing 0-automata no such linear upper bound can exist by exhibiting a series of n-state synchronizing 0-automata whose shortest reset words are of length $\frac{n^2}{4}+o(n^2)$. More precisely, we have the following

Theorem 1. *For each integer $n \geq 8$, there exists a synchronizing 0-automaton A_n with n states and 2 input letters such that the length of the shortest reset word for A_n is $\left\lceil \frac{n^2+6n-16}{4} \right\rceil$.*

In Section 2 we present the construction of the automaton A_n for even n and prove Theorem 1 for this case. The construction for odd n is presented in Section 3 but the corresponding proof (which is pretty similar to the one in the even case) is not included due to space constraints. Section 4 contains some numerical results and a discussion.

2. The automata A_n, n is even

We fix an even number $n = 2m \geq 8$ and let A_{2m} be the DFA $(Q,\{a,b\},\delta)$, where $Q = \{0,\ldots,2m-1\}$, and the transition function δ is defined as follows:

$$\delta(i,a) = \begin{cases} 0 & \text{if } i = 0, \\ i-1 & \text{if } i = 1,\ldots,m-1, \\ 2m-2 & \text{if } i = m, \\ i-1 & \text{if } i = m+1,\ldots,2m-2, \\ 2m-1 & \text{if } i = 2m-1; \end{cases}$$

$$\delta(i,b) = \begin{cases} 0 & \text{if } i = 0, \\ m & \text{if } i = 1,\ldots,m-1, \\ m-1 & \text{if } i = m, \\ 2m-1 & \text{if } i = m+1, \\ i+1 & \text{if } i = m+2,\ldots,2m-3, \\ m+1 & \text{if } i = 2m-2, \\ m+2 & \text{if } i = 2m-1. \end{cases}$$

The automaton is shown in Fig. 2. Clearly, 0 is a zero state in the automaton A_{2m}.

For the sequel, we need some notation. For a word $w \in \{a,b\}^*$, we denote by $|w|$ the length of w and by $w[i]$, where $1 \leq i \leq |w|$, the i^{th} letter in w from the left. If $1 \leq i \leq j \leq |w|$, we denote by $w[i,j]$ the word $w[i]\cdots w[j]$.

For every subset $S \subseteq Q$ and every word $w \in \{a,b\}^*$, we define

$$\delta(S,w) = \bigcup_{q\in S}\{\delta(q,w)\}, \quad \delta(S,w^{-1}) = \{q \in Q \mid \delta(q,w) \in S\}.$$

If $\delta(q,w) = \delta(q',w)$ for two different states $q,q' \in Q$, we say that the word w *merges* the states q and q'.

[1] We were not able to trace the origin of this series of synchronizing automata. It is contained, for instance, in [10] but it should have been known long before [10] since a very close series had appeared already in [9].

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