Information and Computation 243 (2015) 37–49

Contents lists available at ScienceDirect

Information and Computation

www.elsevier.com/locate/yinco

Fast collaborative graph exploration ☆

Dariusz Dereniowski [a], Yann Disser [b], Adrian Kosowski [c,*], Dominik Pająk [d], Przemysław Uznański [e]

[a] Faculty of Electronics, Telecommunications and Informatics, Gdańsk University of Technology, Poland
[b] TU Berlin, Germany
[c] Inria and Université Paris Diderot, France
[d] University of Cambridge, UK
[e] CNRS and Aix-Marseille Université, France

ARTICLE INFO

Article history:
Received 8 September 2013
Available online 11 December 2014

ABSTRACT

We study the following scenario of online graph exploration. A team of k agents is initially located at a distinguished vertex r of an undirected graph. We ask how many time steps are required to complete exploration, i.e., to make sure that every vertex has been visited by some agent.

As our main result, we provide the first strategy which performs exploration of a graph with n vertices at a distance of at most D from r in time $O(D)$, using a team of agents of polynomial size $k = Dn^{1+\epsilon} < n^{2+\epsilon}$, for any $\epsilon > 0$. Our strategy works in the local communication model, in which agents can only exchange information when located at a vertex, without knowledge of global parameters such as n or D.

We also obtain almost-tight bounds on the asymptotic relation between exploration time and team size, for large k, in both the local and the global communication model.

1. Introduction

Exploring an undirected graph-like environment is relatively straightforward for a single agent. Assuming the agent is able to distinguish which neighboring vertices it has previously visited, then in terms of the exploration time, there is no better systematic traversal strategy than a simple depth-first search of the graph, which takes $2(n-1)$ moves in total for a graph with n vertices. The situation becomes more interesting if multiple agents want to collectively explore the graph starting from a common location. If arbitrarily many agents may be used, then we can generously send n^D agents through the graph, where D is the distance from the starting vertex to the most distant vertex of the graph. At each step, we spread out the agents located at each node (almost) evenly among all the neighbors of the current vertex, and thus explore the graph in D steps.

While the cases with one agent and arbitrarily many agents are both easy to understand, it is much harder to analyze the spectrum in between these two extremes. Of course, we would like to explore graphs in as few steps as possible (i.e., close to D), while using a team of as few agents as possible. In this paper we study this trade-off between exploration

☆ Some of the results of this paper appeared in the extended abstract [6], published in the Proceedings of the 40th International Colloquium on Automata, Languages and Programming (ICALP 2013).

* Corresponding author.
E-mail addresses: deren@eti.pg.gda.pl (D. Dereniowski), disser@math.tu-berlin.de (Y. Disser), adrian.kosowski@inria.fr (A. Kosowski), dsp39@cl.cam.ac.uk (D. Pająk), przemyslaw.uznanski@lif.univ-mrs.fr (P. Uznański).

http://dx.doi.org/10.1016/j.ic.2014.12.005
0890-5401/

Table 1
Our bounds for the time required to explore general graphs with using Dn^c agents. The same upper and lower bounds hold for trees. The lower bounds use graphs with $D = n^{o(1)}$.

Communication model	Upper bound	Lower bound
Global communication:	$D \cdot (1 + \frac{1}{c-1} + o(1))$ Theorem 3.3	$D \cdot (1 + \frac{1}{c} - o(1))$ Theorem 4.1
Local communication:	$D \cdot (1 + \frac{2}{c-1} + o(1))$ Theorem 3.3	$D \cdot (1 + \frac{2}{c} - o(1))$ Theorem 4.1

time and team size. A trivial lower bound on the number of steps required for exploration with k agents is $\Omega(D + n/k)$: for example, in a tree, some agent has to reach the most distant node from r, and each edge of the tree has to be traversed by some agent. We look at the case of larger groups of agents, for which D is the dominant factor in this lower bound. This complements previous research on the topic for trees [10,12] and grids [21], which usually focused on the case of small groups of agents (when n/k is dominant).

Another important issue when considering collaborating agents concerns the model that is assumed for the communication between agents. We need to allow communication to a certain degree, as otherwise there is no benefit to using multiple agents for exploration [12]. We may, for example, allow agents to freely communicate with each other, independent of their whereabouts, or we may restrict the exchange of information to agents located at the same location. This paper also studies this tradeoff between global and local communication.

1.1. The collaborative online graph exploration problem

We are given a graph $G = (V, E)$ rooted at some vertex r. The number of vertices of the graph is bounded by n. Initially, a set $\mathcal{A}$ of k agents is located at r. We assume that vertices have unique identifiers that admit a total ordering. In each step, an agent visiting vertex v receives a complete list of the identifiers of the nodes in $N(v)$, where $N(v)$ is the neighborhood of v. Time is discretized into steps, and in each step, an agent can either stay at its current vertex or slide along an edge to a neighboring vertex. Agents have unique identifiers, which allows agents located at the same node and having the same exploration history to differentiate their actions. We do not explicitly bound the memory resources of agents, enabling them in particular to construct a map of the previously visited subgraph, and to remember this information between time steps. An *exploration strategy* for G is a sequence of moves performed independently by the agents. A strategy explores the graph G in t time steps if for all $v \in V$ there exist a time step $s \le t$ and an agent $g \in \mathcal{A}$, such that g is located at v in step s. Our goal is to find an exploration strategy which minimizes the time it takes to explore a graph in the worst case, with respect to the shortest path distance D from r to the vertex furthest from r in the graph. Observe that, given a team of unbounded size, it is possible to perform exploration in D steps (e.g., using a team of n^D agents, and naively flooding the graph, spreading out agents located at a vertex evenly among its neighbors at each step). We look for trade-offs between the size k of the team and the time required for exploration.

We distinguish between two communication models. For exploration *with global communication* we assume that, at the end of each step s, all agents have complete knowledge of the explored subgraph. In particular, in step s all agents know the number of edges incident to each vertex of the explored subgraph which lead to unexplored vertices, but they have no information on any subgraph consisting of unexplored vertices. In exploration *with local communication* two agents can exchange information only if they occupy the same vertex. The information that is exchanged includes the subgraph that an agent explored itself and the information received from other agents prior to the current meeting. Thus, each agent g has its own view on the vertices that were explored so far, based only on the knowledge that originates from the agent's own observations and from other agents that it has met.

1.2. Our results

Our main contribution is an exploration strategy for a team of polynomial size to explore graphs in an asymptotically optimal number of steps. More precisely, for any $\epsilon > 0$, the strategy can operate with $Dn^{1+\epsilon} < n^{2+\epsilon}$ agents and takes time $O(D)$. It works even under the local communication model and without prior knowledge of n or D.

We first restrict ourselves to the exploration of trees (Section 2). We show that with global communication trees can be explored in time $D \cdot (1 + 1/(c-1) + o(1))$ for any $c > 1$, using a team of Dn^c agents. Our approach can be adapted to show that with local communication trees can be explored in time $D \cdot (1 + 2/(c-1) + o(1))$ for any $c > 1$, using the same number of agents. We then carry the results for trees over to the exploration of general graphs (Section 3). We obtain precisely the same asymptotic bounds for the number of time steps that are sufficient to explore graphs with Dn^c agents as for the case of trees, under both communication models. The limit of our approach in terms of the smallest allowed team of agents is a team of $k = (2 + \epsilon)nD$ agents exploring graphs in time $\Theta(D \log n)$, with local communication for any constant $\epsilon > 0$.

Finally, we provide lower bounds for collaborative graph exploration that almost match our positive results (Section 4). More precisely, we show that, in the worst case and for any $c > 1$, exploring a graph with Dn^c agents takes at least $D \cdot (1 + 1/c - o(1))$ time steps in the global communication model, and at least $D \cdot (1 + 2/c - o(1))$ time steps in the local communication model. Table 1 summarizes our upper and corresponding lower bounds.