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On tree automata that certify termination of left-linear term rewriting systems

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Abstract

We present a new method for automatically proving termination of left-linear term rewriting systems on a given regular language of terms. It is a generalization of the match bound method for string rewriting. To prove that a term rewriting system terminates we first construct an enriched system over a new signature that simulates the original derivations. The enriched system is an infinite system over an infinite signature, but it is locally terminating: every restriction of the enriched system to a finite signature is terminating. We then construct iteratively a finite tree automaton that accepts the enriched given regular language and is closed under rewriting modulo the enriched system. If this procedure stops, then the enriched system is compact: every enriched derivation involves only a finite signature. Therefore, the original system terminates. We present two methods to construct the enrichment: roof heights for left-linear systems, and match heights for linear

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systems. For linear systems, the method is strengthened further by a forward closure construction. Using these methods, we give examples for automated termination proofs that cannot be obtained by standard methods.

Keywords: Term rewriting systems; Termination; Forward closures; Rewrite labellings; Tree automata

1. Introduction

We present a new method for approaching the *regular language termination problem*:

- *Given*: A term rewriting system R and a regular language L.
- *Question*: Does R terminate on L, i.e., is there no infinite R-derivation starting from some term in L?

As very special cases, we obtain the uniform termination problem if L is chosen to be the set of all terms over the given signature, and the termination problem on a given starting term t if $L = \{t\}$.

Of course, the method cannot solve every termination problem. It is a semi-algorithm: if successful, it outputs a finite termination certificate that can be checked independently from its construction.

Our method uses tree automata for reachability analysis for certain infinite term rewriting systems. It consists of two steps. In the first step, we switch to an *enrichment* of the given system, i.e., to a rewriting system over a different signature that simulates the original derivations. We consider enriched systems over infinite signatures that are *locally terminating*: every restriction to a finite signature is terminating. In the second step, we compute a *compatible* finite tree automaton for this enrichment, i.e., a tree automaton that contains the enriched given regular tree language and is closed under rewriting modulo the enriched system. The existence of such a compatible automaton ensures that the enriched system is *compact*, i.e., every infinite derivation involves only a finite signature. By local termination of the enrichment, the automaton certifies termination of the original system.

We have previously applied this method to string rewriting [9]. The string rewriting version is implemented in the tools TORPA [23], Matchbox [22] and AProVE [13]. In the present paper, we describe how to extend it to term rewriting. Non-linearities in the rewrite rules complicate both the termination arguments and the automata constructions. The algorithms we present are implemented in Matchbox.

The enrichments that we consider are variants of the original term rewriting system in which the symbols are labelled by natural numbers. Labels start at zero, and in each derivation step, the contractum is labelled uniformly by the successor of the minimum of the labels at certain positions in the redex. By introducing two particular choices for this set of positions, we define *roof heights* and *match heights*. For left-linear systems, bounded roof heights imply termination, while for linear systems, bounded match heights imply termination.

For linear systems, we can improve this criterion. Uniform termination can be concluded from termination on a restricted set of initial terms: the set of right-hand sides of forward closures. We use our method both to compute this set, and to prove termination on it, at the same time. This