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Re-visiting axioms of information systems ☆

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ABSTRACT

Based on the investigation of the axioms used in the theory of information system, the notion of simplified continuous information system is introduced which is considered as a generalization of continuous information systems. Just like the continuous information system, it is also a concrete representation of continuous domains and categorically equivalent to the category of continuous domains and Scott continuous functions. What is more, all types of information systems studied in the literature so far turn out to be special cases of the new type. By defining linear information systems and their entailment functions, we show that the states of a Scott-style information system are exactly the fixpoints of its entailment function. Some axioms about an entailment relation can be characterized by the properties of the corresponding entailment function, and in particular, the simplified continuous information systems are solely determined by the idempotence of entailment functions.

1. Introduction

The classical framework for the denotational semantics of programming language is the category of Scott domains with continuous functions. In order to provide a less abstract and better accessible approach to domain theory, as early as 1982, Scott [1] introduced the notion of information system (more explicitly, the algebraic information system), which has some essential features of a logic calculus, as a concrete representation of Scott domains. Technically, there is an equivalence between the category *aIS* of algebraic information systems and the category *SD* of Scott domains (Larsen and Winskel [2], 1984). The logical flavor of information systems makes them suitable for other purposes, such as Formal Concept Analysis [3,4] and Chu Spaces [5].

Following Scott's idea, many authors devised several kinds of information systems such that some subclasses of domains are reconstructed successfully. Hoofman [6] generalized Scott's information systems to the continuous case, but his continuous information systems capture only bounded-complete continuous domains (continuous Scott domains). Based on Hoofman's representation, Bedregal and Silva [7] developed a continuous logical system and analyzed the logical aspects of a continuous Scott domain. The notion of prime information system introduced by Zhang [8], only catches hold of DI-domains. With observing that transitiveness and interpolation play fundamental roles of construction of domains, Vickers

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[9] built certain information systems for reconstruction of continuous posets. Though Vickers' approach is more general and can be used to represent all continuous domains, it is not Scott-style.

Recently, Spreen, Xu and Mao [10] defined a certain type of continuous information systems when analyzing how abstract bases and their continuous information systems (instead of Hoofman [6]) correspond to each other. Furthermore, they showed that the category of continuous information systems is equivalent to the category of continuous domains. The approach in [10] not only belongs to the Scott-style, but is able to recover all continuous domains.

It is well-known that both Scott domains and bounded-complete continuous domains are special cases of continuous domains. Also, algebraic information systems are precisely contained in Hoofman's continuous information systems. However, as is pointed out by Example 36 in [6], the continuous information system defined by Hoofman isn't a proper subclass of the continuous information systems presented in [10]. One of the main motivations of this paper is to look for a type of information systems which not only includes all Scott-style information systems considered in the literature, but precisely captures continuous domains, i.e., the new type is still as expressive as continuous information systems in [10]. Following the work [10], we omit the axiom of entailment consistency and present a new type of continuous information system, named simplified continuous information system, the category of which is equivalent to that of continuous domains. Though we choose the approximate mappings in [10], the proof of equivalence is completely different from [10].

Viewing from data mining, states in an information system are essentially some data mined from the set of tokens by using a special entailment relation. The dynamic data mining methods based on functions are mathematically better than those on relations. Another motivation of this paper is to provide the fixpoint description of state which is a key notion in information system theory, and to characterize some axioms of entailment relations which are extensively used in the literature. For these purposes, we introduce linear information systems and their entailment functions. All states in a linear information system are proved to be the fixpoints of its entailment function. Moreover, some axioms about an entailment relation can be described by the properties of the corresponding entailment function, and in particular, the simplified continuous information systems are determined by the idempotence of the entailment function.

The paper is organized as follows: Section 2 recalls some basic notions and results of domain theory. Section 3 introduces simplified continuous information systems and analyzes the connection between continuous domains and them. The equivalence between the category of simplified continuous information systems and the category of continuous domains are obtained. The similar result also holds for the algebraic case. Section 4 discusses how the states in a Scott-style information system and the fixpoints of its Scott continuous entailment function correspond to each other. The functional characterization of some axioms of entailment relations in information systems are also considered in this section. As is expected, idempotent Scott continuous entailment functions fully capture simplified continuous information systems.

2. Domains

For the convenience of readers, we recall some notions in domain theory in this section. Let P be a poset. A nonempty subset D of P is said to be directed if for two arbitrary elements d_1 and d_2 of D, there exists a $d_3 \in D$ such that $d_1 \leq d_3$ and $d_2 \leq d_3$. A directed complete poset simply denoted by *dcpo* is a poset such that each directed set of it has the least upper bound. Let $x, y \in P$, x is said to approximate y (in symbol $x \ll y$) if and only if for every directed set $D \subseteq P$, $y \leq \bigvee D$ means that there is a $d \in D$ such that $x \leq d$. x is said to be compact if $x \ll x$. (We use κ_P to denote the set of all compact elements of P.) Let $\Downarrow x = \{a \mid a \in P, a \ll x\}$ for every $x \in P$. A subset β of P is said to be a basis of P if for every $x \in P$, $\Downarrow x \cap \beta$ is a directed set and $x = \bigvee(\Downarrow x \cap \beta)$. A *dcpo* is called a domain (or a continuous domain) if it has a basis. In particular, a *dcpo* is called an algebraic domain if all compact elements of it form a basis. As is well known, a *dcpo* P is an algebraic domain if and only if for every $x \in P$, there is a directed set $D(x) \subseteq \kappa_P$ with $x = \bigvee D(x)$.

A function f between two posets P_1 and P_2 is called Scott continuous if f preserves all joins of directed sets, that is, for a directed set D of P_1, the existence of $\bigvee D$ implies $f(\bigvee D) = \bigvee f(D)$. *DOMAIN* denotes the category of domains with Scott continuous functions. The algebraic domain category *ALG* with algebraic domains as objects is the full subcategory of *DOMAIN*.

Definition 2.1. An abstract basis is given by a set B together with a transitive relation $\prec$ on B such that

$$\text{(INT)} \qquad M \prec x \Rightarrow \exists y \in B. M \prec y \prec x$$

holds for all elements x and nonempty finite subset M of B.

A round ideal A of an abstract basis $< B, \prec >$ is a nonempty subset of B such that A is directed with respect to the relation $\prec$ and for every $x \in A$, $y \prec x$ implies $y \in A$. Let $IDL(B)$ be the set of all round ideals ordered by inclusion. With the tool of round ideals, abstract bases have a typical application to domain theory:

Theorem 2.1. *Let P be a domain and β be a basis of P. Then:*

(1) *$< \beta, \ll >$ is an abstract basis*
(2) *$IDL(\beta)$ is isomorphic to P.*