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No Future without *(a hint of)* Past
A Finite Basis for 'Almost Future' Temporal Logic

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ABSTRACT

Kamp's theorem established the expressive completeness of the temporal modalities Until and Since for the First-Order Monadic Logic of Order (*FOMLO*) over real and natural time flows. Over natural time, a single future modality (Until) is sufficient to express all future *FOMLO* formulas. These are formulas whose truth value at any moment is determined by what happens from that moment on. Yet this fails to extend to real time domains: here no finite basis of future modalities can express all future *FOMLO* formulas. In this paper we show that finiteness can be recovered if we slightly soften the requirement that future formulas must be totally past-independent: we allow formulas to depend just on the arbitrarily recent past, and maintain the requirement that they be independent of the rest – actually – of most of the past. We call them 'almost future' formulas, and show that there is a finite basis of almost future modalities which is expressively complete (over all Dedekind complete time flows) for the almost future fragment of *FOMLO*.

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1. Introduction

Temporal Logic (*TL*) introduced to Computer Science by Pnueli in [6] is a convenient framework for reasoning about "reactive" systems. This made temporal logics a popular subject in the Computer Science community, enjoying extensive research in the past 30 years. In *TL* we describe basic system properties by *atomic propositions* that hold at some points in time, but not at others. More complex properties are expressed by formulas built from the atoms using Boolean connectives and *Modalities* (temporal connectives): A k-place modality M transforms statements $\varphi_1 \dots \varphi_k$ possibly on 'past' or 'future' points to a statement $\mathsf{M}(\varphi_1 \dots \varphi_k)$ on the 'present' point t_0. The rule to determine the truth of a statement $\mathsf{M}(\varphi_1 \dots \varphi_k)$ at t_0 is called a *Truth Table*. The choice of particular modalities with their truth tables yields different temporal logics. A temporal logic with modalities $\mathsf{M}_1, \dots, \mathsf{M}_k$ is denoted by $TL(\mathsf{M}_1, \dots, \mathsf{M}_k)$.

The simplest example is the one place modality $\mathsf{F}X$ saying: "X holds some time in the future". Its truth table is formalized by $\varphi_{\mathsf{F}}(t_0, X) \equiv (\exists t > t_0)X(t)$. This is a formula of the First-Order Monadic Logic of Order (*FOMLO*) – a fundamental formalism in Mathematical Logic where formulas are built using atomic propositions $P(t)$, atomic relations between elements $t_1 = t_2$, $t_1 < t_2$, Boolean connectives and first-order quantifiers $\exists t$ and $\forall t$. Most modalities used in the literature are defined by such *FOMLO* truth tables, and as a result every temporal formula translates directly into an equivalent *FOMLO* formula. Thus, the different temporal logics may be considered a convenient way to use fragments of *FOMLO*. *FOMLO* can

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also serve as a yardstick by which to check the strength of temporal logics: A temporal logic is *expressively complete* for a fragment L of *FOMLO* if every formula of L with a single free variable t_0 is equivalent to a temporal formula.

Actually, the notion of expressive completeness is with respect to the type of the underlying model since the question whether two formulas are equivalent depends on the domain over which they are evaluated. Any (partially) ordered set with monadic predicates is a model for *TL* and *FOMLO*, but the main, *canonical*, linear time intended models are the naturals $\langle \mathbb{N}, < \rangle$ for discrete time and the reals $\langle \mathbb{R}, < \rangle$ for continuous time.

A major result concerning *TL* is Kamp's theorem [5,2], which states that the pair of modalities "*X until Y*" and "*X since Y*" is expressively complete for *FOMLO* over the above two linear time canonical models.

Many temporal formalisms studied in computer science concern only future formulas – whose truth value at any moment is determined by what happens from that moment on. For example the formula *X until Y* says that X will hold from now (at least) until a point in the future when Y will hold. The truth value of this formula at a point t_0 does not depend on the question whether $X(t)$ or $Y(t)$ hold at earlier points $t < t_0$.

Over the discrete model $\langle \mathbb{N}, < \rangle$ Kamp's theorem holds also for *future formulas* of *FOMLO*: The future fragment of *FOMLO* has the same expressive power as *TL*(Until) [3,2]. The situation is radically different for the continuous time model $\langle \mathbb{R}, < \rangle$. In [4] it was shown that *TL*(Until) is not expressively complete for the future fragment of *FOMLO* and there is no easy way to remedy it. In fact it was shown in [4] that there is no temporal logic with a finite set of future modalities which is expressively equivalent to the future fragment of *FOMLO* over the reals.

The proof there goes (roughly) as follows: define a sequence of future formulas $\phi_i(z)$ such that given any set B of modalities definable in the future fragment of *FOMLO* by formulas of quantifier depth at most n, the formula $\phi_{n+1}(z)$ is not expressible in *TL*(B).

The interesting point is that these formulas are all expressible in a temporal language based on the future modality Until plus the modality K^- of [2]. The formula $\mathsf{K}^-(P)$ holds at a time point t_0 if given any 'earlier' t, no matter how close, we can always come up with a t' in between ($t < t' < t_0$) where P holds. This is of course not a future modality: the formula $\mathsf{K}^-(P)$ is past-dependent. And it turns out that not only the above mentioned sequence of future formulas – but all future formulas – can be expressed (over real time) in *TL*(Until, K^-). This is a consequence of Gabbay's separation theorem [2].

This future-past mixture of Until and K^- is somewhat better than the standard Until-Since basis in the following sense: Although K^- is (like Since) a past modality, it does not depend on much of the past: The formula $\mathsf{K}^-(P)$ depends just on an arbitrarily short 'recent past', and is actually independent of most of the past. In this sense we may say that it is an "almost future" formula (see Section 3.1 for precise definitions).

In [4] it was conjectured that *TL*(Until, K^-) is expressively complete for almost future formulas of *FOMLO*. Our main result here affirms this conjecture with respect to all Dedekind complete time domains. An extended abstract concerning the particular case of real time domains was published in [7].

The rest of the paper is organized as follows: In Section 2 we recall the definitions of the monadic logic, the temporal logics and Kamp's theorem. In Section 3.1 we define "almost futureness" and make a trivial small step towards the proof. Most of the 'machinery' needed for the proof is in Sections 3.2 and 3.3, with the heart of the proof in Lemma 3.25. Section 3.4 then just puts it all together to complete the proof. Finally, Section 4 states further results and comments.

2. Preliminaries

We start with the basic definitions of First-Order Monadic Logic of Order (***FOMLO***) and Temporal Logic (***TL***), and some well-known results concerning their expressive power. Fix a ***signature*** (finite or infinite) $\mathcal{S}$ of ***atoms***. We use $P, Q, R, S \dots$ to denote members of $\mathcal{S}$. Syntax and semantics of both logics are defined below with respect to such a fixed signature.

2.1. First-order monadic logic of order

Syntax: In the context of *FOMLO*, the atoms of $\mathcal{S}$ are referred to (and used) as ***unary predicate symbols***. Formulas are built using these symbols, plus two binary relation symbols, $<$ and $=$, and a finite set of ***first-order variables*** (denoted by $x, y, z, \dots$). Formulas are defined by the grammar:

$$\textit{atomic} ::= \; x < y \;\mid\; x = y \;\mid\; P(x) \qquad \text{(where } P \in \mathcal{S}\text{)}$$

$$\varphi ::= \; \textit{atomic} \;\mid\; \neg\varphi_1 \;\mid\; \varphi_1 \vee \varphi_2 \;\mid\; \varphi_1 \wedge \varphi_2 \;\mid\; \exists x \varphi_1 \;\mid\; \forall x \varphi_1$$

The notation $\varphi(x_1, \dots, x_n)$ implies that φ is a formula where the x_i's are the only variables that may occur free; writing $\varphi(x_1, \dots, x_n, P_1, \dots, P_k)$ additionally implies that the P_i's are the only predicate symbols that may occur in φ. We will also use the standard abbreviated notation for ***bounded quantifiers***, e.g.: $(\exists x)_{>z}(\dots)$ denotes $\exists x((x > z) \wedge (\dots))$, $(\forall x)^{\leq z}(\dots)$ denotes $\forall x((x \leq z) \rightarrow (\dots))$, $(\forall x)^{<u}_{>l}(\dots)$ denotes $\forall x((l < x < u) \rightarrow (\dots))$, etc. Finally, as usual, ***True(x)*** denotes $P(x) \vee \neg P(x)$ and ***False(x)*** denotes $P(x) \wedge \neg P(x)$.

Semantics: Formulas are interpreted over *structures*. A ***structure*** over $\mathcal{S}$ is a triplet $\mathcal{M} = (\mathcal{T}, <, \mathcal{I})$ where $\mathcal{T}$ is a set – the ***domain*** of the structure, $<$ is an irreflexive partial order relation on $\mathcal{T}$, and $\mathcal{I} : \mathcal{S} \rightarrow \mathcal{P}(\mathcal{T})$ is the ***interpretation*** of the atoms in the structure (where $\mathcal{P}$ is the powerset notation). We use the standard notation $\mathcal{M}, t_1, t_2, \dots t_n \models \varphi(x_1, x_2, \dots x_n)$.

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