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Axiomatisation and decidability of multi-dimensional Duration Calculus

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Abstract

The Shape Calculus is a spatio-temporal logic based on an n-dimensional Duration Calculus tailored for the specification and verification of mobile real-time systems. After showing non-axiomatisability, we give a complete embedding in n-dimensional interval temporal logic and present two different decidable subsets, which are important for tool support and practical use.

Keywords: Real-time systems; Mobile systems; Spatial logic; Temporal logic; Duration Calculus

1. Introduction

1.1. Motivation

Mobile real-time systems are omnipresent today, e.g., in airplane and railroad control systems. Failures in these systems may have severe consequences which can even endanger lives. Formal specification and automatic verification are promising approaches to increase the safety of such systems. However, for these systems real-time aspects as well as spatial aspects are important.

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Thus, commonly used formalisms that concentrate on either timing or spatial behaviour fall short in these cases because they need to abstract from important issues.

A pivotal task in the UniForM [32] project in cooperation with the industrial partner Elpro was the development of a control for a single-tracked line segment (SLS) for tramways. The problem is to ensure the safety of trams if only one track is available and this track is passed in both directions and occupied by up to two trams simultaneously as long as they head into the same direction. A controller has been derived, simulated, and partially verified using techniques for real-time systems, namely PLC-Automata [19]. However, the main safety requirement, i.e., mutual exclusion of trams with opposite directions on the critical section, is a spatio-temporal property and cannot be expressed in purely time-dependent models like PLC Automata [18].

Similar problems arise in the specifications of mobile robots [42]. Each robot itself constitutes a real-time system, whereas the specification of the overall system behaviour has additional spatial requirements, for example collision avoidance.

1.2. Research contributions

The shortcomings described above led us to the idea to extend the Duration Calculus, a well-known formalism for real-time systems, with proven applicability [28], to be able to describe also spatial properties. The use of the formalism is similar to the use of the original Duration Calculus when no spatial reasoning is required. Thus, experienced users of temporal logics can easily adopt the new features.

We present the Shape Calculus[1] (SC), a spatio-temporal logic based on the Duration Calculus, extending the results in [42] and [43]. Shape Calculus is interval based and possesses an integral operator $\int$ for measuring time as well as space. We elaborate that this formalism is well suited for the application domain of mobile real-time systems. We present four major results for this formalism. First, we prove that the full logic is undecidable and non axiomatisable, even for discrete infinite models of time and space. To this end, we present a reduction of its validity problem to the emptiness problem of tiling languages. The full real-time logic Duration Calculus is known to be non-axiomatisable for continuous temporal domains, but still decidable for a subset in the discrete setting which shows to be undecidable in the multi-dimensional case. This relates to the undecidability of multi-dimensional products of decidable modal logics as discussed in [24]. Second, we present an axiomatisation of Shape Calculus relatively to an n-dimensional interval logic without the $\int$ operator, a result similar to the one for continuous time Duration Calculus. In practice, acceptance of formal methods is increased dramatically by tool support. Hence, we discuss decidable subsets of the Shape Calculus. Our third contribution is a decidable subset of Shape Calculus based on results for discrete Duration Calculus. There the subset assumes a discrete and infinite temporal domain but finite spatial domains. The decision procedure reduces validity to emptiness of regular languages. This subset has already led to a prototypical implementation of a model checker [38]. Forth, we elaborate a decidable subset of discrete Shape Calculus using ideas from logic combination [24] and relating a syntactical subset of Shape Calculus without chop alternation to fusions of Duration Calculus. This approach proceeds by reducing validity to iteratively checking emptiness of regular languages.

[1] The name Shape Calculus was proposed by A. Ravn during a presentation of early ideas.