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Logics of communication and change

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Abstract

Current dynamic epistemic logics for analyzing effects of informational events often become cumbersome and opaque when common knowledge is added for groups of agents. Still, postconditions involving common knowledge are essential to successful multi-agent communication. We propose new systems that extend the epistemic base language with a new notion of 'relativized common knowledge', in such a way that the resulting full dynamic logic of information flow allows for a compositional analysis of all epistemic postconditions via perspicuous 'reduction axioms'. We also show how such systems can deal with factual alteration, rather than just information change, making them cover a much wider range of realistic events. After a warm-up stage of analyzing logics for public announcements, our main technical results are expressivity and completeness theorems for a much richer logic that we call LCC. This is a dynamic epistemic logic whose static base is propositional dynamic logic (PDL), interpreted epistemically. This system is capable of expressing all model-shifting operations with finite action models, while providing a compositional analysis for a wide range of informational events. This makes LCC a serious candidate for a standard in dynamic epistemic logic, as we illustrate by analyzing some complex communication scenarios, including sending successive emails with both 'cc' and 'bcc' lines, and other private announcements to subgroups. Our proofs involve standard modal techniques, combined with a new application of Kleene's theorem on finite automata, as well as new Ehrenfeucht games of model comparison.

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1. Introduction

Epistemic logic deals with what agents consider possible given their current information. This includes knowledge about facts, but also *higher-order information* about information that other agents have. A prime example is *common knowledge*. A formula φ is common knowledge if everybody knows φ, everybody knows that everybody knows that φ, and so on. *Common belief* is an important related notion. Indeed, although this paper is mainly written in 'knowledge' terminology, everything we say also holds, with minor technical modifications, when describing agents' *beliefs*, including common belief.

Dynamic epistemic logics analyze changes in both basic and higher-order information. One of the main attractions of such systems is their transparent analysis of effects of communicative actions in the format of an equivalence between epistemic postconditions and preconditions. A typical example concerns knowledge of an agent after and before a public announcement:

$$[\varphi]\Box_a\psi \leftrightarrow (\varphi \rightarrow \Box_a[\varphi]\psi).$$

This axiom says that after the announcement that φ agent a knows that ψ iff φ implies that agent a knows that after φ is announced ψ will be true. We call such principles *reduction axioms*, because the announcement operator is 'pushed through' the epistemic operator, in such manner that on the right hand side the complexity of the formula in the scope of the announcement is less that the complexity of the formula in the scope of the announcement on the left hand side. This reduction axiom describes the interaction between the announcement operator and the epistemic operator. If there is a reduction axiom for each logical operator in the language, such a set of axioms make logical systems particularly straightforward. For instance, the logic of public announcements without common knowledge has an easy completeness proof by way of a translation that follows the reduction axioms. Formulas with announcements are translated to provably equivalent ones without announcements, and completeness follows from the known completeness of the epistemic base logic. Thus, the dynamic logic of the announcement operator is fully characterized by the reduction axioms.

This is the technical way of putting things. But more importantly, reduction axioms like the one above also reflect a desirable *methodology*: they allow for *compositional analysis* of the epistemic effects of informational events. This is particularly helpful with more complex scenarios, where it is not at all easy to describe just what agents should know, or not, after some communication has taken place: say, a round of emails involving both public 'cc' and half-private 'bcc' lines. A dynamic epistemic logic with a complete set of reduction axioms has an ideal 'harmony' between its static and dynamic parts allowing for complete compositional analysis. So, it is worth finding such systems whenever they exist. Finally, more locally, specific reduction axioms also express interesting