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Approximate strip packing: Revisited [☆]

Xin Han [a,b,1], Kazuo Iwama [c], Deshi Ye [d], Guochuan Zhang [d,*,2]

[a] Software School, Dalian University of Technology, Dalian 116620, China
[b] Key Lab for Ubiquitous Network and Service Software of Liaoning Province, China
[c] RIMS, Kyoto University, Kyoto 606-8502, Japan
[d] College of Computer Science, Zhejiang University, Hangzhou 310027, China

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ABSTRACT

In this paper we establish an algorithmic framework between bin packing and strip packing, with which strip packing can be very well approximated by applying some bin packing algorithms. More precisely we obtain the following results: (1) Any off-line bin packing algorithm can be applied to strip packing maintaining almost the same asymptotic worst-case ratio. (2) A class of Harmonic-based algorithms for bin packing, such as Refined Harmonic, Modified Harmonic, Harmonic++, can be applied to online strip packing. In particular, we show that online strip packing admits an upper bound of $1.58889 + \epsilon$ on the asymptotic competitive ratio, for any arbitrarily small $\epsilon > 0$. This significantly improves the previously best bound of 1.6910 and affirmatively answers an open question posed by Csirik and Woeginger (1997). Moreover, the time complexity mainly depends on a sorting procedure and the bin packing algorithms employed.

1. Introduction

In strip packing a set of rectangles with widths and heights both upper bounded by 1, is packed into a strip with width 1 and infinite height. Rectangles must be packed such that no two rectangles overlap with each other and the sides of the rectangles are parallel to the strip sides. Rotations are not allowed. The objective is to minimize the height of the strip needed to pack all the given rectangles. If we know all rectangles before constructing a packing, then this problem is *offline*. In contrast, in *online* strip packing rectangles are coming one by one and a placement decision for the current rectangle must be done before the next rectangle appears. Once a rectangle is packed its placement is permanent.

It is well known that strip packing is a generalization of bin packing. Namely if we restrict all input rectangles to be of the same height, then strip packing can be regarded as bin packing. Thus any negative results for bin packing remain true for strip packing. More precisely, strip packing is NP-hard in the strong sense and the asymptotic lower bound 1.5401 [16] is valid for online strip packing.

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* Corresponding author.
E-mail addresses: hanxin.mail@gmail.com (X. Han), iwama@kuis.kyoto-u.ac.jp (K. Iwama), yedeshi@zju.edu.cn (D. Ye), zgc@zju.edu.cn (G. Zhang).

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Previous results For the offline version Coffman et al. [4] presented the algorithms NFDH (Next Fit Decreasing Height) and FFDH (First Fit Decreasing Height), and showed that the respective asymptotic worst-case ratios are 2 and 1.7. Golan [6] and Baker et al. [2] found better algorithms with worst case ratios of 4/3 and 5/4, respectively. Using linear programming and random techniques, an asymptotic fully polynomial time approximation scheme (AFPTAS) was given by Kenyon and Rémila [9]. For the online version Baker and Schwarz [3] introduced an online strip packing strategy called a shelf algorithm. A shelf is a rectangular part of the strip with width one and height at most one so that (i) every rectangle is either completely inside or completely outside of the shelf and (ii) every vertical line through the shelf intersects the interior of at most one rectangle. Shelf packing is an elegant idea to exploit bin packing algorithms. By employing bin packing algorithms *Next Fit* and *First Fit*, Baker and Schwarz [3] obtained the asymptotic competitive ratios arbitrarily close to 2 and 1.7, respectively. This idea was extended to the Harmonic shelf algorithm by Csirik and Woeginger [5], making the asymptotic competitive ratio arbitrarily close to $h_\infty \approx 1.6910$. Moreover it was shown that h_∞ is the best upper bound a shelf algorithm can achieve, no matter what online bin packing algorithm is used. In the late 80s and early 90s, online bin packing algorithms with asymptotic competitive ratios better than h_∞ were presented [10–12,17]. However, it was not known whether these online algorithms could be adapted to the online strip packing problem [5].

The core of shelf packing is reducing the two-dimensional problem to the one-dimensional problem. Basically shelf algorithms consist of two steps. The first one is *shelf design* which only takes the heights of rectangles into account. One shelf can be regarded as a bin with a specific height. The second step is *packing into a shelf*, where rectangles with similar heights are packed into the same shelves. This step is done by employing some bin packing algorithm to pack the rectangles with a total width upper bounded by one into a shelf. Clearly, to maintain the quality of bin packing algorithms in shelf packing we must improve the first step.

Our contribution We propose a batch packing strategy and establish a general algorithmic framework between bin packing and strip packing. It is shown that any offline bin packing algorithm can be used for offline strip packing maintaining almost the same asymptotic worst-case ratio. As an example, the well known bin packing algorithm FFD can be adapted to approximate strip packing with an asymptotic worst-case ratio of $11/9 + \epsilon$, for any $\epsilon > 0$, where the running time is $O(n \log n)$. A simple asymptotic fully time approximation scheme (AFPTAS) for strip packing can be derived by our techniques using the AFPTAS of [8] for bin packing as a black box.

For the online case, we can apply the Super Harmonic algorithms [13] to online strip packing. It implies that the known Harmonic-based bin packing algorithms [10–13] can be converted into online strip packing algorithms. In particular, strip packing admits an online algorithm with asymptotic performance bound of $1.58889 + \epsilon$, for any given $\epsilon > 0$, by employing Seiden's Harmonic++ algorithm [13], which is the current champion for online bin packing. Our result affirmatively answers the open question in [5]

Main ideas Recall that strip packing becomes bin packing if all rectangles have the same height. This motivates us to convert the strip packing problem into the bin packing problem by constructing a set of *new* rectangles called *boxes* with the same height by bundling a subset of items. Then we just call the algorithm in the bin packing problem to pack the generated boxes into the strip. More precisely, in the offline case, we pack in batch the rectangles with similar width on top of each other into rectangular boxes of pre-specified height of c, where $c > 1$ is a sufficiently large constant. Then we obtain a set of new rectangles (boxes) of the same height. The next step is to use bin packing algorithms on the new set. In the on-line case the strategy is slightly different. We divide the rectangles into two groups according to their widths, to which we apply the above batching strategy and the standard shelf algorithms, respectively.

Asymptotic worst-case (competitive) ratio To evaluate online or offline approximation algorithms for strip packing and bin packing we use the standard measure defined as follows.

Given an input list L and an approximation algorithm A, we denote by $OPT(L)$ and $A(L)$, respectively, the height of the strip (the number of bins) used by an optimal offline algorithm and the height of the strip (the number of bins) used by algorithm A for packing list L.

The *asymptotic worst-case ratio* R_A^∞ of algorithm A is defined by

$$R_A^\infty = \limsup_{n\to\infty} \max_L \{A(L)/OPT(L) | OPT(L) = n\}.$$

For online algorithms, the asymptotic worst-case ratio is also referred as the "competitive ratio".

2. The offline problem

Given a rectangle R, throughout the paper, we use $w(R)$ and $h(R)$ to denote its width and height, respectively.

Fractional strip packing A fractional strip packing of L is a packing of any list L_f obtained from L by subdividing some of its rectangles by horizontal cuts: a rectangle (w, h) can be replaced by a sequence $(w, h_1), (w, h_2), \ldots, (w, h_k)$ of rectangles such that $h = \sum_{i=1}^{k} h_i$.

The following lemma is from Section 3.1 of the reference paper [9].