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Bounds for online bin packing with cardinality constraints ☆

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ABSTRACT

We study a bin packing problem in which a bin can contain at most k items of total size at most 1, where $k \geq 2$ is a given parameter. Items are presented one by one in an online fashion. We analyze the best absolute competitive ratio of the problem and prove tight bounds of 2 for any $k \geq 4$. Additionally, we present bounds for relatively small values of k with respect to the asymptotic competitive ratio and the absolute competitive ratio. In particular, we provide tight bounds on the absolute competitive ratio of First Fit for $k = 2, 3, 4$, and improve the known lower bounds on asymptotic competitive ratios for multiple values of k. Our method for obtaining a lower bound on the asymptotic competitive ratio using a certain type of an input is general, and we also use it to obtain an alternative proof of the known lower bound on the asymptotic competitive ratio of standard online bin packing.

1. Introduction

We study a variant of bin packing called bin packing with cardinality constraints (BPCC). In this problem, the input consists of items, denoted by $1, 2, \ldots, n$, such that item i has a size $s_i > 0$ associated with it, and there is a global parameter $k \geq 2$, called the cardinality constraint. The goal is to partition the input items into subsets, called bins, such that the total size of items of every bin is at most 1, and the number of items packed into each bin does not exceed k. We believe that bounding the number of items as well as their total size provides a more accurate model for packing problems; for example, a data center can usually only store a constant number of files. BPCC is a well-studied variant in the offline and online environments [17,18,16,5,1,8,10,11].

In this paper, we study online algorithms that receive and pack input items one by one, without any information on future input items. A fixed optimal offline algorithm that receives the complete list of items before packing it is denoted by OPT. For an input I and algorithm ALG, we let $ALG(I)$ denote the number of bins that A uses to pack I. We also use $OPT(I)$ to denote the number of bins that OPT uses for a given input I. The absolute competitive ratio of an algorithm ALG is the supremum ratio over all inputs I between the number of bins $ALG(I)$ that it uses and the number of bins $OPT(I)$

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that OPT uses. The asymptotic competitive ratio is the limit of absolute competitive ratios R_K when K tends to infinity and R_K takes into account only inputs for which OPT uses at least K bins. Note that (by definition), for a given algorithm (for some online bin packing problem), its asymptotic competitive ratio never exceeds its absolute competitive ratio. If the algorithm is offline, the standard terms are *approximation ratio* and *asymptotic approximation ratio*. For an algorithm whose competitive ratio (or approximation ratio) does not exceed R, we say that it is an R-competitive (or R-approximation). We see a bin as a set of items, and for a bin B, we let $s(B) = \sum_{i \in B} s_i$ be its level or load.

Bin packing problems are often studied with respect to the asymptotic measures. Approximation algorithms were designed for the offline version of BPCC (that is strongly NP-hard for $k \geq 3$) [17,16,5,10], and the problem has an asymptotic fully polynomial approximation scheme (AFPTAS) [5,10]. Using elementary bounds, it was shown by Krause, Shen, and Schwetman [17] that the cardinality constrained variant of First Fit (FF), that packs an item i into a minimum indexed bin where it fits both with respect to size and cardinality (i.e., the target bin must have at most $k-1$ items and its current level must be at most $1 - s_i$), has an asymptotic competitive ratio of at most $2.7 - \frac{2.4}{k}$. For $k \to \infty$, the asymptotic competitive ratio of FF is at most 2.7, which follows from the result of [17] and also from that of [12], since this is a special case of vector bin packing (with two dimensions).

Next, we survey other known results for BPCC. The case $k = 2$ is solvable using matching techniques in the offline scenario, but it is not completely resolved in the online scenario. Liang [20] showed a lower bound of $\frac{4}{3}$ on the asymptotic competitive ratio for this case, Babel et al. [1] improved the lower bound to $\sqrt{2} \approx 1.41421$, and designed an algorithm whose asymptotic competitive ratio is at most $1 + \frac{1}{\sqrt{5}} \approx 1.44721$ (improving over the previous bound, which was proved for FF). Recently, Fujiwara and Kobayashi [11] improved the lower bound to 1.42764. For larger k, there is a 2-competitive algorithm [1], and improved algorithms are known for $k = 3, 4, 5, 6$ (whose competitive ratios are at most 1.75, 1.86842, 1.93719, and 1.99306, respectively) [8].

Note that the upper bound of [17] for FF and $k = 3$ is 1.9, and an algorithm whose competitive ratio is at most 1.8 was proposed by [1]. A full analysis of the cardinality constrained variant of the Harmonic algorithm [19] is given in [8], and its competitive ratios for $k = 2$ and $k = 3$ are 1.5 and $\frac{11}{6}$, respectively (its competitive ratio is in [2, 2.69103] for $k \geq 4$). As for lower bounds, until recently, except for the case $k = 3$ for which a lower bound of 1.5 on the competitive ratio was proved in [1], most of the known lower bounds followed from the analysis of lower bounds for standard bin packing [28,26,2].

New lower bounds for many values of k were given by Fujiwara and Kobayashi in [11], and in particular, they proved lower bounds of 1.5 and $\frac{25}{17} \approx 1.47058$ for $k = 4$ and $k = 5$, respectively. For $6 \leq k \leq 9$, the current best lower bound remained 1.5, which was implied by the lower bound of Yao [28], and for $k = 10$ and $k = 11$, lower bounds of $\frac{80}{53} \approx 1.50943$ and $\frac{44}{29} \approx 1.51724$, respectively, were proved in [11] (see [11] for the lower bounds of other values of k). In this paper we provide improved lower bounds on the asymptotic competitive ratio of arbitrary online algorithms for $k = 5, 7, 8, 9, 10, 11$. The values of these lower bounds are 1.5 for $k = 5$, and approximately 1.51748, 1.5238, 1.5242, 1.526, 1.5255, for $k = 7, 8, 9, 10, 11$, respectively. We also provide improved lower bounds for larger values of k, see Table 1.

The methods used in this work differ from the ones used before. In the past, the inputs that were used for $k \geq 3$ consisted of sub-inputs with identical items, such that the number of items of each kind is equal (and the input may be stopped after a sub-input was presented). We use this framework too, but the numbers of items will not necessarily be equal. Moreover, unlike the constructions given in the previous work, in some cases we do not adapt previously used sequences (those that were used for proving lower bounds for standard online bin packing [26,2]), but we use a different input sequence. To avoid dealing with packing patterns (subsets of items that can fit into bins), we use weight functions. For that, we provide a general theorem that allows to deal with inputs coming from a class of inputs, and is useful for proving lower bounds on the asymptotic competitive ratio for various bin packing problems (we demonstrate it also on standard online bin packing, getting the lower bound of [2]).

There are few known results for the absolute measures. The asymptotic $(1 + \varepsilon)$-approximation algorithm of Caprara, Kellerer, and Pferschy [5] uses $(1 + \varepsilon)OPT(I) + 1$ bins to pack the items of an input I, and thus, choosing $\varepsilon > 0$ to be small (for example $\varepsilon = \frac{1}{100}$) results in a polynomial time absolute $\frac{3}{2}$-approximation algorithm. This is the best possible unless P = NP. In the online environment, it is not difficult to see that given the absolute upper bound of 1.7 on the competitive ratio of FF for standard bin packing [7], the upper bound of $2.7 - 2.4/k$ becomes an absolute one (we provide the proof here for completeness). In this paper, we also analyze the absolute competitive ratio, and show a tight bound of 2 on the absolute competitive ratio for any $k \geq 4$, and a tight bound of 1.5 for $k = 2$. The upper bound for $k = 4$ is proved for FF. An upper bound for $k = 5$ is proved using an algorithm that performs FF except for one case. We show that a variant of the algorithm of [1] has an absolute competitive ratio 2 for any $k \geq 3$. In the case $k = 3$, we provide a lower bound of $\frac{7}{4} = 1.75$ on the absolute competitive ratio of any algorithm, and show that the absolute competitive ratio of FF is $\frac{11}{6} \approx 1.8333$. A complete analysis of the *asymptotic* competitive ratio of FF can be found in [6].

For standard bin packing [25,13–15,19,28,21], it is known that the asymptotic competitive ratio is in [1.5403, 1.58889] [2,22], and the absolute competitive ratio is $\frac{5}{3}$ [29,3] (the absolute competitive ratio of FF without cardinality constraints is 1.7 [7]). Another related problem is called *class constrained bin packing* [9,23,24,27]. In that problem each item has a color, and a bin cannot contain items of more than k colors (for a fixed parameter k). BPCC is the special case of that problem where all items have distinct colors (with the same value of k). A lower bound of 2 on the asymptotic competitive ratio was proved for the former problem [24] even for the case where items have equal sizes, whereas the latter problem (BPCC) has an algorithm whose asymptotic competitive ratio is 2 [1] (and BPCC is trivial for the case of equal item sizes). Moreover, for