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On the displacement for covering a unit interval with randomly placed sensors

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ABSTRACT

Consider n mobile sensors placed independently at random with the uniform distribution on a barrier represented as the unit line segment $[0, 1]$. The sensors have identical sensing radius, say r. When a sensor is displaced on the line a distance equal to d it consumes energy (in movement) which is proportional to some (fixed) power $a > 0$ of the distance d traveled. The energy consumption of a system of n sensors thus displaced is defined as the sum of the energy consumptions for the displacement of the individual sensors.
We focus on the problem of energy efficient displacement of the sensors so that in their final placement the sensor system ensures coverage of the barrier and the energy consumed for the displacement of the sensors to these final positions is minimized in expectation. In particular, we analyze the problem of displacing the sensors from their initial positions so as to attain coverage of the unit interval and derive trade-offs for this displacement as a function of the sensor range. We obtain several tight bounds in this setting thus generalizing several of the results of [10] to any power $a > 0$.

1. Introduction

One of the most important problems in sensor networks is minimizing battery consumption when accomplishing various tasks such as monitoring an environment, tracking events along a barrier and communicating. In this study, the environment being considered consists of a line segment barrier (which for simplicity is set to the unit interval $[0, 1]$), while the accompanying monitoring problem investigated is ensuring coverage of the barrier in the sense that every point in the line segment is within the range of a sensor.

We consider the case where the sensors are equipped with omnidirectional sensing antennas of identical range $r > 0$; thus a sensor placed at location x in the unit interval can sense any point at distance at most r either to the left or right of x. The initial placement of the sensors does not guarantee barrier coverage since the sensors have been placed initially independently at random with the uniform distribution on a barrier. To attain coverage of the line segment it is required to displace the sensors from their original locations to new positions on the line while at the same time taking into account their sensing range r. Further, for some fixed constant $a > 0$ if a sensor is displaced a distance d the energy consumed by this sensor is considered to be proportional to d^a. More generally, for a set of n sensors, if the ith sensor is displaced a distance d_i, for $i = 1, 2, \ldots, n$, then the energy consumed by the whole system of n sensors is $\sum_{i=1}^{n} d_i^a$. In this paper we

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study the minimum total (or sum) energy consumption (in expectation) in the movement of the sensors so as to attain coverage of the unit segment when the energy consumed per sensor is proportional to some (fixed) power of the distance traveled. The present study generalizes some known results (see [10]) on the sensor displacement for $a = 1$ to arbitrary $a > 0$. Motivation for the extended model being proposed is that the energy consumption induced by individual sensor displacement may not be linear in this displacement, but rather be dependent on some power of the distance traversed. Further, the parameter a in the exponent may well represent various conditions of the surface of the barrier, e.g., friction, lubrication, etc., which may affect the overall energy consumption of the sensor system.

1.1. Related work

There is extensive literature about area and barrier (also known as perimeter) coverage by a set of sensors (e.g., see [1,3,15,12,14,5]). The coverage problem for planar domains with pre-existing anchor (or destination) points was introduced in [4]. The deterministic version of the sensor displacement problem on a linear domain (or interval) was introduced in [6]. Several optimization variants of the displacement problem were considered. The complexity of finding an algorithm that optimizes the displacement depends 1) on the types of the sensors, 2) the type of the domain, and 3) whether one is minimizing the sum or maximum of the sensor movements. For the unit interval the problem of minimizing the sum is NP-complete if the sensors may have different ranges but is in polynomial time when all the sensor ranges are identical [7]. The problem of minimizing the maximum is NP-complete if the region consists of two intervals [6] but is polynomial time for a single interval even when the sensors may have different ranges [5]. Related work on deterministic algorithms for minimizing the total and maximum movement of sensors for barrier coverage of a planar region may be found in [4].

More importantly, our work is closely related to the work of [10] where the authors consider the expected minimum total displacement for establishing full coverage of a unit interval for n sensors placed uniformly at random. Our analysis and problem statement generalizes some of the work of [10] from $a = 1$ to all exponents $a > 0$. A comprehensive study of sensor displacement to arbitrary probability distributions using techniques from queueing theory can be found in the forthcoming [11].

1.2. Outline and results of the paper

Our work generalizes some of the work of [10] to the more general setting when the cost of movement is proportional to a fixed power of the distance displacement.

The overall organization of the paper is as follows. In Section 2 we provide several basic combinatorial facts that will be used in the sequel. In Section 3 we prove combinatorially how to obtain tight bounds when the range of the sensors is $r = \frac{1}{2n}$. We show that the expected sum of displacement to the power a is

$$\frac{\left(\frac{a}{2}\right)!}{2^{\frac{a}{2}}(1+a)}\frac{1}{n^{\frac{a}{2}-1}} + O\left(\frac{1}{n^{\frac{a}{2}}}\right),$$

when a is an even positive number, and in

$$\Theta\left(\frac{1}{n^{\frac{a}{2}-1}}\right),$$

when a is an odd natural number. In Section 4 we prove the occurrence of threshold whereby the expected minimum sum of displacements to the power a (a is positive natural number) remains in $\Theta\left(\frac{1}{n^{\frac{a}{2}-1}}\right)$ provided that $r = \frac{1}{2n} + \frac{f(n)}{2}$, where $f(n) > 0$ and $f(n) = o(n^{-3/2})$. In Section 5 we study the more general version of the sensors movement to the power a, where $a > 0$ and $r > \frac{1}{2n}$. If $r \geq \frac{6}{2n}$ we first present the Algorithm 1 that uses expected

$$O\left(\frac{1}{n^{\frac{a}{2}-1}}\left(\frac{\ln n}{n}\right)^{\frac{a}{2}}\right)$$

total movement to power a, where $a > 0$. Finally, Section 6 provides the conclusions.

2. Basic facts

In this section we recall some known facts about special functions and special numbers which will be useful in the analysis in the next sections. The Euler Beta function (see [13])

$$\mathrm{B}(c,d) = \int_0^1 x^{c-1}(1-x)^{d-1}dx \tag{1}$$

is defined for all complex numbers c, d such as $\Re(c) > 0$ and $\Re(d) > 0$. Moreover, for positive integer numbers c, d we have

$$\mathrm{B}(c,d)^{-1} = \binom{c+d-1}{c}c \tag{2}$$

Let us define a function $g_{c:d}(x) = x^{c-1}(1-x)^{d-1}$ on the interval $[0, 1]$. We say that a random variable $X_{c,d}$ concentrated on the interval $[0, 1]$ has the $\mathrm{B}(c, d)$ distribution with parameters c, d if it has the probability density function $f(x) = (\mathrm{B}(c, d))^{-1}x^{c-1}(1-x)^{d-1}$. Hence,

$$\Pr[X_{c,d} < t] = \frac{1}{\mathrm{B}(c,d)}\int_0^t g_{c:d}(x)dx \tag{3}$$

We will use the following notations for the rising and falling factorial respectively [9]

$$n^{\overline{k}} = \begin{cases} 1 & \text{for } k = 0 \\ n(n+1)\dots(n+k-1) & \text{for } k \geq 1, \end{cases}$$

$$n^{\underline{k}} = \begin{cases} 1 & \text{for } k = 0 \\ n(n-1)\dots(n-(k-1)) & \text{for } k \geq 1. \end{cases}$$

Let $\left[{n \atop k}\right]$, $\left\{{n \atop k}\right\}$ be the Stirling numbers of the first and second kind respectively, which are defined for all integer