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Zhang neural networks for a set of linear matrix inequalities with time-varying coefficient matrix [☆]

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ABSTRACT

Zhang neural networks (ZNN) model is developed for solving a set of time-varying linear matrix inequalities, referred to as Stein matrix inequality, which exploits the time-derivative information of time-varying coefficients. Computer simulation results show that the proposed ZNN model is efficient and superior for such kind of linear matrix inequalities (LMIs).

1. Introduction

In recent decades, online matrix inequalities problems are widely encountered in numerous science and engineering applications [1]. For example, the following linear matrix inequality (LMI),

$$AXB + X \leqslant C, \tag{1}$$

where $A \in \mathbb{R}^{m\times m}$, $B \in \mathbb{R}^{n\times n}$ and $C \in \mathbb{R}^{m\times n}$ are given constant matrices, and $X \in \mathbb{R}^{m\times n}$ unknown, is usually as an essential part in many applications such as control system design, optimization and signal processing [2]. In view of these, it is considered to solve such kind of LMIs, referred to as Stein matrix inequality for the equivalence in (1) is a Stein matrix equation. Obviously, if A, B are nonsingular and $B^{-1} = A^{\mathrm{T}}$, post-multiplying by B^{-1} can yield the Lyapunov matrix inequality.

There are two general types of well-developed solutions to the matrix inequality problems [1]. One is to transform the matrix inequalities into optimization problems and solved by classical methods. The other is based on iterative methods. However, when the system is of large-scale, both of them may not be effective and lead to slow convergence [3–5]. With the development of artificial neural networks theory, more and more authors have begun to develop neural networks models to solve such problems [1,3,4,6–9]. And as a powerful approach, the neural-dynamic model based on recurrent neural network is proposed. The classical two models are the gradient/gradient-based neural networks (GNN) [8] and Zhang neural networks (ZNN) [1,3,4,10]. GNN is designed for solving the static/time-invariant systems, while ZNN is for time-varying cases (that is A, B and C vary with $t \geqslant 0$).

In this paper, a ZNN model is investigated for online solution of Stein matrix inequality (1) with time varying, i.e.,

$$A(t)X(t)B(t) + X(t) \leqslant C(t). \tag{2}$$

By introducing a time-varying matrix with each element greater than or equal to zero, the time-varying Stein matrix inequality (2) is converted to a time-varying Stein matrix

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equation. Then, the ZNN model is developed for solving the converted time-varying matrix equation, thus the original time-varying matrix inequality. Theoretical analysis and numerical results are presented to demonstrate the excellent performance of the proposed ZNN approach for the time-varying Stein matrix inequality (2).

The rest of this paper is organized as follows. In Section 2, preliminaries for time-varying Stein LMI and results related are introduced. Section 3 presents the ZNN model for solving the converted time-varying linear matrix equation and the original time-varying linear matrix inequality. In Section 4, computer simulation results are illustrated. Section 5 is the concluding remarks.

2. Preliminary

In this section, the problem formulation of time-varying Stein matrix inequality is presented first. Then, the conversion from Stein matrix inequality to Stein matrix equation is proposed by introducing a time-varying matrix.

2.1. Problem

The following problem of time-varying Stein matrix inequality is considered,

$$A(t)X(t)B(t) + X(t) \leqslant C(t), \tag{3}$$

where $A(t) \in \mathbb{R}^{m\times m}$, $B(t) \in \mathbb{R}^{n\times n}$ and $C(t) \in \mathbb{R}^{m\times n}$ are smoothly time-varying given matrices whose time-derivatives are known numerically or could be estimated accurately, and $X(t) \in \mathbb{R}^{m\times n}$ is the time-varying unknown matrix to be solved. The objective is to find $X(t)$ such that the time-varying Stein LMI (3) holds for any time $t \geqslant 0$.

2.2. Converted Stein matrix equation

To transform a time-varying Stein matrix inequality to a time-varying Stein matrix equation, the time-varying Stein LMI (3) is reformulated as follows:

$$F(X(t),t) = A(t)X(t)B(t) + X(t) - C(t) \leqslant \mathbf{0}, \tag{4}$$

where each element of $F(x(t),t)$ is less than or equal to zero. Thus, introducing a time-varying matrix $\Lambda^{\cdot 2}(t) \in \mathbb{R}^{m\times n}$ whose element is greater than or equal to zero leads to the following time-varying Stein matrix equation,

$$A(t)X(t)B(t) + X(t) - C(t) + \Lambda^{\cdot 2}(t) = \mathbf{0}, \tag{5}$$

where superscript $^{\cdot 2}$ denotes the square of the element of a matrix and $\Lambda(t) \in \mathbb{R}^{m\times n}$ is also an unknown matrix to be obtained.

To solve (5), the following related definitions and lemmas are needed.

Definition 2.1 *([11])*. Given matrices $\tilde{A} = (\tilde{a}_{ij}) \in \mathbb{R}^{m\times n}$ and $\tilde{B} = (\tilde{b}_{ij}) \in \mathbb{R}^{p\times q}$, the Kronecker product (or direct product or tensor product) of $\tilde{A}$ and $\tilde{B}$, denoted by $\tilde{A}\otimes\tilde{B}$, is defined as the following block matrix,

$$\tilde{A}\otimes\tilde{B} = \begin{pmatrix} \tilde{a}_{11}\tilde{B} & \cdots & \tilde{a}_{1n}\tilde{B} \\ \vdots & \ddots & \vdots \\ \tilde{a}_{m1}\tilde{B} & \cdots & \tilde{a}_{mn}\tilde{B} \end{pmatrix} \in \mathbb{R}^{mp\times nq}.$$

Note that, in general, $\tilde{A}\otimes\tilde{B} \neq \tilde{B}\otimes\tilde{A}$, and $\tilde{A}\otimes\tilde{A}^{\mathrm{T}} \neq \tilde{A}^{\mathrm{T}}\otimes\tilde{A}$ except $\tilde{A} = \tilde{A}^{\mathrm{T}}$.

Definition 2.2 *([11])*. Given a matrix $\tilde{C} = (\tilde{c}_{ij}) \in \mathbb{R}^{m\times n}$, $\mathrm{vec}(\tilde{C})$ is defined to be the mn-vector formed by stacking the columns of $\tilde{C}$ as follows,

$$\mathrm{vec}(\tilde{C}) = (\tilde{c}_{11}, \tilde{c}_{21}, \cdots, \tilde{c}_{m1}, \tilde{c}_{12}, \tilde{c}_{22}, \cdots, \tilde{c}_{m2}, \cdots, \tilde{c}_{1n}, \tilde{c}_{2n}, \cdots, \tilde{c}_{mn})^{\mathrm{T}} \in \mathbb{R}^{mn}.$$

Lemma 2.1 *([2]). The time-varying Stein matrix equation (5) is uniquely solvable, if $\lambda_i(A(t)) \cdot \lambda_j(B(t)) \neq -1$ for $\forall i = 1,2,3,\cdots,m$ and $\forall j = 1,2,3,\cdots,n$ at any time instant $t \in [0,+\infty)$, where $\lambda_i(P(t))$ denotes the ith eigenvalue of the time-varying matrix $P(t)$.*

Lemma 2.2 *([2]). If Lemma 2.1 is satisfied, then $M(t) := B^{\mathrm{T}}(t)\otimes A(t) + I$ is a nonsingular time-varying matrix, where I denotes an appropriately-dimensioned identity matrix.*

Then, by solving the time-varying Stein matrix equation (5), a time-varying solution $X(t)$ and a time-varying matrix $\Lambda(t)$ can be obtained. With the previous analysis, the following inequality is obtained,

$$A(t)X(t)B(t) + X(t) - C(t) = -\Lambda^{\cdot 2}(t) \leqslant \mathbf{0},$$

which indicates that the solution $X(t)$ of (5) is also the time-varying solution of the time-varying Stein LMI (3), that is to say, LMI (3) can be solved via the online solution of (5).

3. ZNN model for time-varying Stein LMI

In this section, the Zhang neural networks (ZNN) model is presented to solve time-varying matrix equation (5) thus time-varying Stein LMI (3).

First, the following matrix-valued indefinite error-function is constructed,

$$E(t) = A(t)X(t)B(t) + X(t) - C(t) + \Lambda^{\cdot 2}(t). \tag{6}$$

Then, the ZNN model can be established as follows [4],

$$\dot{E}(t) = \frac{dE(t)}{dt} = -\Gamma\Phi(E(t)), \tag{7}$$

where the matrix-valued design parameter Γ could be simply γI with constant scalar $\gamma > 0$, which is used to scale the convergence rate of the solution, and $\Phi(\cdot): \mathbb{R}^{m\times n} \longrightarrow \mathbb{R}^{m\times n}$ denotes a matrix-valued activation function array of neural networks. In general, the function $\phi(\cdot)$, element of $\Phi(\cdot)$, can be any monotonically increasing odd activation function, such as

- the linear activation function,

  $\phi(e) = e;$
- the hyperbolic-sine (h-s) activation function (with $\xi = 2$),

  $$\phi(e) = \sinh(e) = \frac{\exp(\xi e) - \exp(-\xi e)}{2};$$